PROGRESSION IN WRITTEN CALCULATIONS

This document aims to provide guidance and ensure consistency in mathematical written methods and approaches to calculation across years 3-6 at Bure Valley School.

Guidance has been arranged according to year group expectations, although it may be appropriate for teachers to make reference to lower year group guidance in order to meet the needs of all the children in the class.

Although the 2014 curriculum does not specify the approach or method children should use when calculating, we have identified methods that the children will be taught across the junior phase; these are in line with our cluster calculation policy.

It is important that children are equipped with the skills to solve problems they are presented with and they are able to apply these skills to real life contexts. As part of the teaching, children need to be taught how to select the best method for the calculation.

The hierarchy of thinking to decide on a method should be:

Can I do it in my head?

Can I use jottings to help me?

Should I use a written method?

Should I use a calculator?

Aims of the document

The Progression in Written Calculations document aims to:

- Ensure pupils are equipped with the necessary calculation skills for their age
- Develop teachers' understanding of how and when to move children on to calculation methods
- Ensure that teachers know how to support children who may not yet be at their age related expectation
- Encourage pupils to decide on the most efficient method to use when solving a calculation
- Promote the use of concrete and pictorial representations for calculations and demonstrate how these can be used
- Ensure a smooth transition between year groups

Rationale

Lower KS2

When children start the junior phase they should already have a secure understanding of the four operations. They will continue to build on the concrete and conceptual understanding they have already gained in KS1 to develop their arithmetical confidence. In addition and subtraction, children are taught methods based on place value; this is a skill which supports both mental and written calculations. The use of larger numbers means that children need to be provided with the skills to move away from 'counting in ones' or using finger-based methods. A large focus at the beginning of KS2 is on multiples and near multiples of 10, 100 and 1000, where children are able to use complementary addition as an accurate means of achieving fast and accurate answers. Formal methods such as column addition and subtraction are introduced; initially children are not required to cross boundaries where they may 'carry' or 'exchange' to or from other columns. Column methods will also be presented in expanded forms to ensure place value is deep-rooted in their thinking. During this part of the key stage, multiplication and division facts are thoroughly learnt, memorised and consolidated up to 12x12. Efficient written methods are introduced for multiplying and dividing 2 or 3 digit numbers by a single-digit number, as are efficient mental strategies. For example, when multiplying or dividing by 4, children will be encouraged to double and double again, or half and half again, respectively; this relates the understanding of fractions and helps to secure this understanding. Similarly, when multiplying or dividing by numbers such as 5 or 20, children will be encouraged to build on their understanding of performing the same operation using 10. The concept of decimal number is introduced at this stage also; children cement a firm understanding of one-place decimals, multiplying and dividing them by 10 and 100.

Upper KS2

The most notable change between lower and upper KS2 calculation is the size and complexity of the numbers presented. Children move on from dealing with mainly whole numbers to performing operations with both decimals and fractions. The consolidation of written methods will continue with children solving problems with numbers that include up to six digits and numbers with up to two decimal places. Children's mental strategies are developed as well as formal written methods. Children move from expanded written methods to more compact methods (this is only done when the teachers is confident that a child has secure enough understanding of place value). Practise of efficient and flexible mental strategies enables children to solve calculations even when the numbers are large, e.g. the answer to 800 x 70000 can be derived by strong times table and place value understanding. In addition to this, it is in Year 5 and 6 that children are moved onto written algorithms of multiplication and division. The grid method that children have been introduced to in previous years moves onto column multiplication, while short division is a new concept introduced to children in Year 5 followed by long division when the teacher feels that it is appropriate for the child to do so. As well as the algorithms being consolidated, children practise applying the four operations to fractions, decimals and negative numbers.

Addition

Add numbers up to 3 digits

Y3

Use partitioning method to add two or three 3-digit numbers or three 2-digit numbers. Begin to use expanded column addition to add numbers with three digits, move onto compact if children are ready and have secure place value understanding.

Partitioning

Partition both numbers and recombine.

or

Count on by partitioning the second number only e.g.

247 + 125 = 247 + 100 + 20+ 5

= 347 + 20 + 5

= 367 + 5

= 372

Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.

Towards a Written Method

Introduce expanded column addition modelled with place value counters (Dienes could be used for those who need a less abstract representation). See below for example.

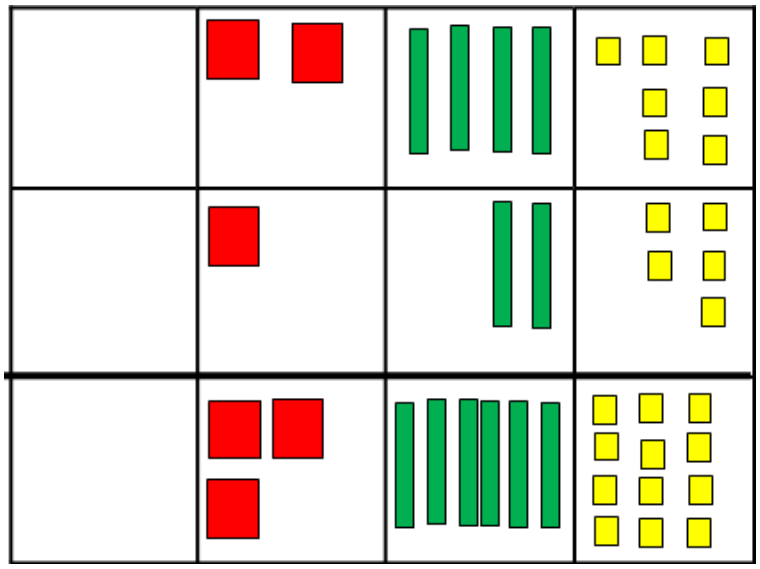
247 + 125 = 372

$$\begin{array}{r} 200 + 40 + 7 \\ \underline{100 + 20 + 5} \\ 300 + 60 + 12 = 372 \end{array}$$

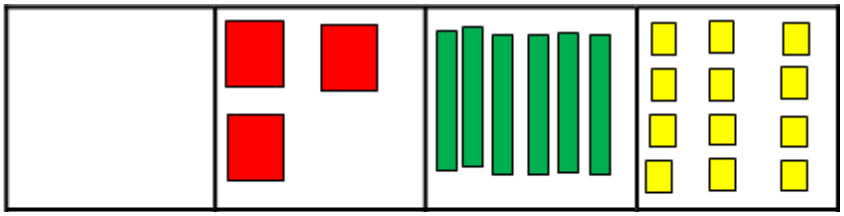
$$\begin{array}{rll} & 247 & \\ +& \underline{125} & \\ & 12 & (7+2) \\ & 60 & (40+20) \\ & \underline{300} & (200+100) \\ & 372 & \end{array}$$

Children can transfer their understanding of partitioning to the column method. To begin with, they should write the calculation next to the answer (as seen on the left) so that they are showing the steps. This step can be dropped when children show secure understanding.

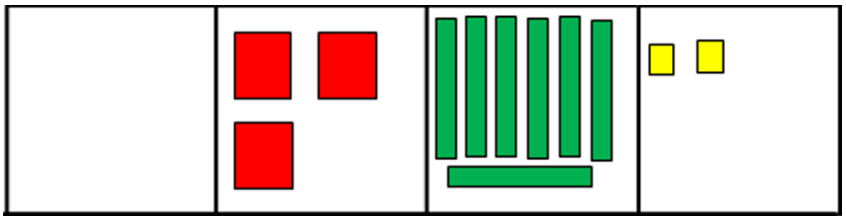
Visual representation



Leading to children understanding the exchange between tens and ones.



3 hundreds, 6 tens and 12 ones becomes 3 hundreds 7 tens and 2 ones, once exchanged:



Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method. Children who are very secure and confident with the compact method should be moved on to using numbers which involved carrying. ***N.B carrying should be placed underneath the calculation.***

Addition

Add numbers up to 4 digits

Y4

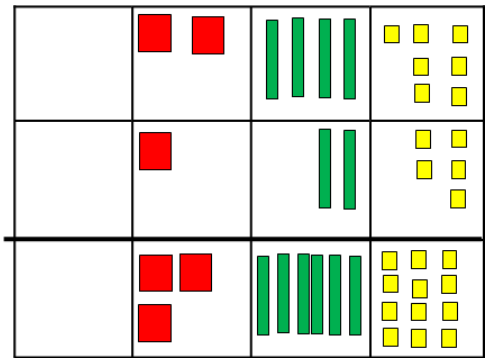
Continue to use the compact method, adding ones first and carrying underneath the calculation. Also include money measures context.

Mental methods should continue to develop, supported by a range of models and images. The bar model should continue to be used to help with problem solving. See separate document addressing mental methods for further guidance.

Written methods (progressing to 4-digits)

Expanded column addition modelled with place value counters, progressing to calculations with 4-digit numbers.

247 + 125 = 372



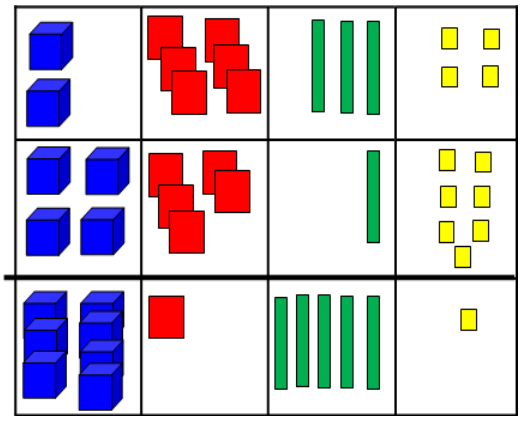
200 + 40 + 7
+ 100+ 20 + 5
300 + 60 + 12 = 372

247
+125
12
60
300
372

Compact written method

Extend to numbers with at least four digits.

2634 + 4517 = 7151



2634
+ 4517
7151
1 1

N.B. Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty.

Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).

$$\begin{array}{r} 72.8 \\ +\ 54.6 \\ \hline 127.4 \\ {\scriptstyle 1\quad 1} \end{array}$$

Addition

Add numbers more than 4 digits

Y5

Continue to use the compact method, adding ones first and carrying underneath the calculation. Extend to larger numbers and decimals numbers including money measures in context.

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. See separate document addressing mental methods for further guidance. Children should practise with increasingly large numbers to aid fluency. E.g. 12462 + 2300 = 14762

Written methods (progressing to more than 4-digits)

As year 4, progressing when understanding of the expanded method is secure, children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm. Children should use abstract only methods when the teacher knows that their understanding is secure. Ensure calculations are presented to children in a linear format as well as in a column: this ensures that children are able to write calculations into columns if not already done so for them.

```
 172.83
+ 54.68
 227.51
1    1  1
```

Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers.

Addition

Y6

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Children should be confident in using the column method, with most using the compact column method. Some children may need visual aids.

Mental methods should continue to develop, supported by a range of models and images. The bar model should continue to be used to help with problem solving. See separate document addressing mental methods for further guidance.

Written methods

As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured.
Continue calculating with decimals, including those with different numbers of decimal places. Ensure calculations are presented to children in a linear format as well as in a column: this ensures that children are able to write calculations into columns if not already done so for them.

Problem Solving

Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding. Encourage the use of bar model as a representative for these problems. NB: The bar model should not be seen as a method, it is simply a way in which a problem can be represented.

Subtraction

Y3

Subtract numbers up to 3 digits

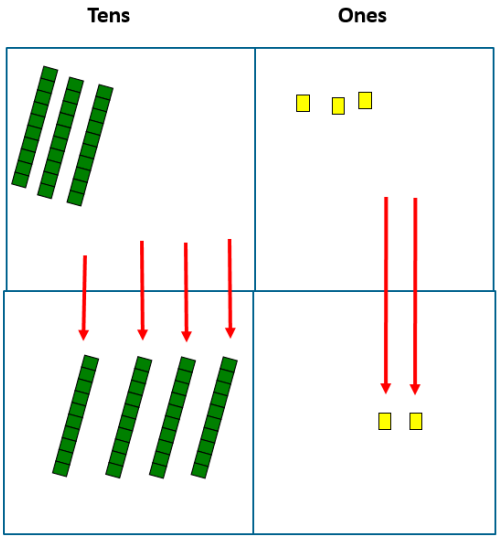
Use the number line to model subtraction and difference questions of two or three 3-digit numbers or three 2-digit numbers. Introduce expanded column subtraction with no decomposition, modelled with dienes equipment.

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving (see Y1 and Y2).

Children should make choices about whether to use complementary addition or counting back, depending on the numbers involved.

Written methods (progressing to 3-digits)

Introduce expanded column subtraction with no decomposition, modelled with Dienes equipment.



$$\begin{array}{rr} 70 & 5 \\ -\,40 & 2 \\ \hline 30 & 3 \end{array}$$

A number line and expanded column method may be compared next to each other.

Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

Subtraction

Y4

Subtract numbers up to 4 digits

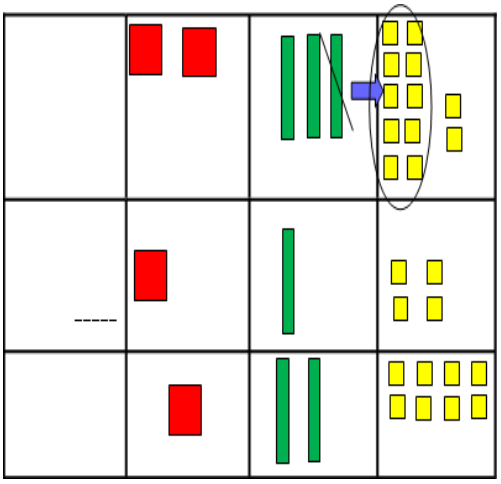
Children to continue using the number line to find the difference until they have a secure understanding of how to use the expanded column subtraction (without decomposition – see Y3). When children are secure with expanded column subtraction, introduce decomposition.

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods (progressing to 4-digits)

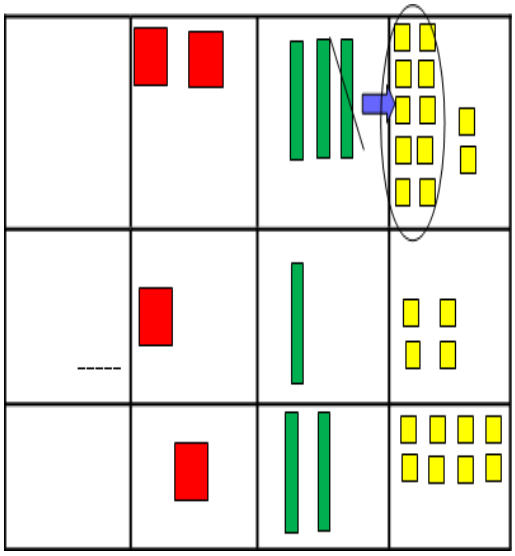
Expanded column subtraction with decomposition, modelled with place value counters, progressing to calculations with 4-digit numbers.

https://www.youtube.com/watch?v=NXCsEkMLWtY



$$\begin{array}{rrrr} & & \overset{20}{\cancel{30}} & \overset{1}{2} \\ - & 100 & 10 & 4 \\ \hline & 100 & 10 & 8 \end{array}$$

If understanding of the expanded method is secure, children will move on to the formal method of decomposition, which again can be initially modelled with dienes equipment (See next page).



$$\begin{array}{r} {\scriptstyle 2\quad 1} \\ 2\not{3}2 \\ -\,114 \\ \hline 118 \\ \hline \end{array}$$

Subtraction

Y5

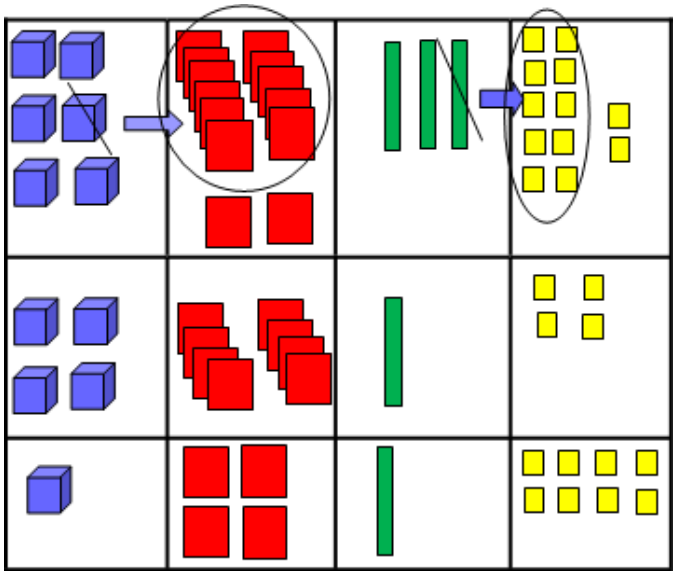
Subtract numbers more than 4 digits

Continue to use column subtraction, both with and without decomposition. Only a small number of children should still be reliant on the number line method. Pupil should move on when their understanding is secure. Extend to larger numbers and decimals numbers including

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods (progressing to more than 4-digits)

When understanding of the expanded method is secure, children will move on to the formal method of decomposition, which can be initially modelled with dienes equipment.



$$\begin{array}{r} {\scriptstyle 5\ \ 1\ \ 2\ \ 1} \\ 6\cancel{2}\cancel{3}2 \\ -\ 4814 \\ \hline 1418 \end{array}$$

Progress to calculating with decimals, including those with different numbers of decimal places

Subtraction

Y6

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Children should be confident in using the column method, with most using the compact column method. Some children may need visual aids.

Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Written methods

As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with decomposition to be secured.

Teachers may also choose to introduce children to other efficient written layouts which help develop conceptual understanding. For example:

326
-148
-2
-20
200
178

Continue calculating with decimals, including those with different numbers of decimal places.

Multiplication

Y3

Write and calculate multiplication using multiplication tables, including for two-digit times one-digit numbers

Children should be able to recall multiplication and division facts of 3, 4 and 8 multiplication tables. They should use partitioning to multiply basic two-digit times one-digit numbers and then use this knowledge of partitioning to transfer to an array (see written methods section).

Mental methods

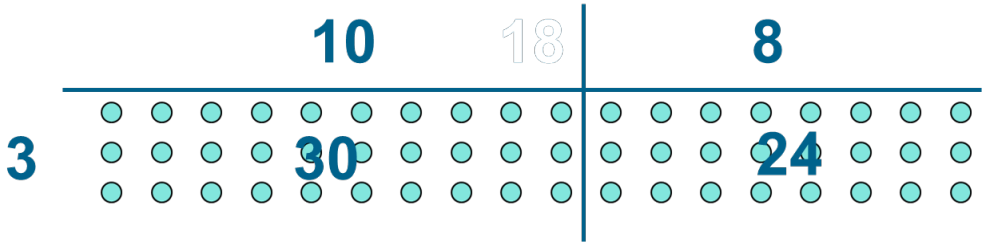
Doubling 2 digit numbers using partitioning.

Demonstrating multiplication on a number line – jumping in larger groups of amounts

13 x 4 = 10 groups 4 = 3 groups of 4

Written methods (progressing to 2d x 1d)

Developing written methods using understanding of visual images. Use counters to develop an array of the multiplication – this may start with 18 x 3 as 18 rows of 3; this will also show proportionality to the numbers which are being calculated. Discuss how the numbers could be portioned and then, using straws, divide the sections to begin the process of transferring to the grid method (see below).



This should be developed into the grid method:

	10	8
3	30	24

Give children opportunities for them to explore this and deepen understanding using Dienes apparatus and place value counters.

Multiplication

Y4

Write and calculate multiplication using multiplication tables, including for two-digit and three-digit times one-digit and two-digit numbers.

Children should be able to recall multiplication and division facts for multiplication tables up to 12 x 12. They should use partitioning to multiply basic two-digit times one-digit numbers and then use this knowledge of partitioning to transfer to an array and then a formal algorithm (see written methods section).

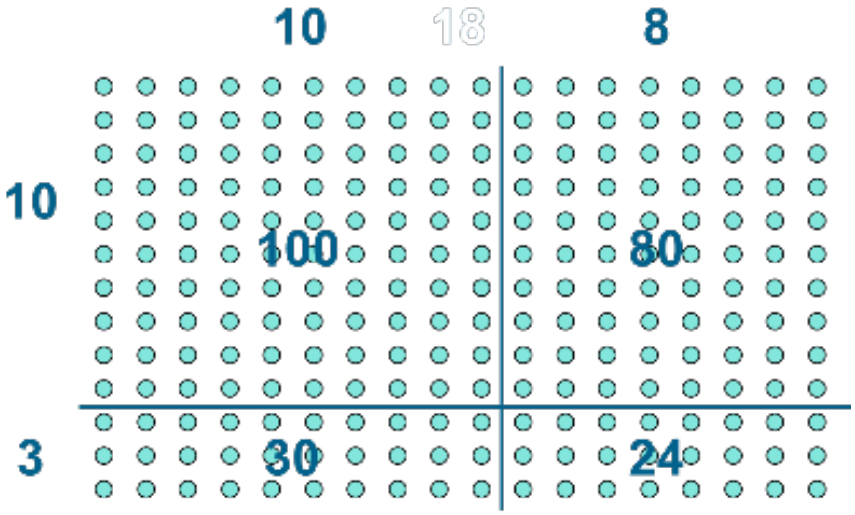
Mental methods

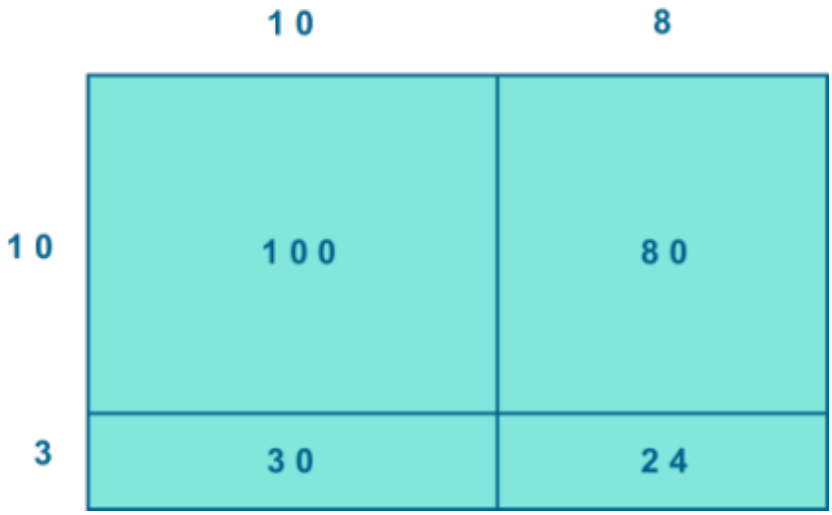
Counting in multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.

Solving practical problems where children need to scale up. Relate to known number facts. (E.g. how tall would a 25cm sunflower be if it grew 6 times taller?)

Written methods (progressing to 3d x 2d)

Children to embed and deepen their understanding of the grid method to multiply up 2d x 2d. Ensure this is still linked back to their understanding of arrays and place value counters.





Once children are secure with the grid method, they can be introduced to a more formal (expanded) algorithm. Initially they should expand their multiplication like the example below. Children can 'drop' the multiplication in brackets once they are secure with the process. Children should draw on their formal addition methods understanding to find the final answer.

$$
\begin{array}{rl}
 & 13 \\
\times & 7 \\
\hline
 & 21 \quad (7 \times 3) \\
 & 70 \quad (7 \times 10) \\
\hline
 & 91
\end{array}
$$

Multiplication

Y5

Write and calculate multiplication using multiplication tables, including for up to four-digit by two-digit numbers.

Children should be able to recall multiplication and division facts for multiplication tables up to 12 x 12. They continue to use partitioning to multiply basic two-digit times one-digit numbers and then use this knowledge of partitioning to transfer to a formal algorithm (see written methods section).

Mental methods

X by 10, 100, 1000 using moving digits ITP

Use practical resources and jottings to explore equivalent statements (e.g. 4 x 35 = 2 x 2 x 35)

Recall of prime numbers up 19 and identify prime numbers up to 100 (with reasoning)

Solving practical problems where children need to scale up. Relate to known number facts.

Identify factor pairs for numbers

Written Methods

Children will begin year 5 using both the grid method and the expanded formal method. Children should continue using grid method until they are secure enough to move on to the formal algorithm. Children should continue expanding the formal written method until they are ready (see Year 4) moving onto 2dx2d and 3dx2d, up to 4dx2d.

$$
\begin{array}{rl}
 & 24 \\
\times & 38 \\
\hline
 & 32 \quad (8 \times 4) \\
 & 160 \quad (8 \times 20) \\
 & 120 \quad (30 \times 4) \\
 & 600 \quad (30 \times 20) \\
\hline
 & 912 \\
 & \overset{1}{\cancel{}}
\end{array}
$$

Secure timestable fluency is a big advantage to children in this method. They will be able to quickly calculate 30 x 20 because they are secure with 3 x 2 and therefore know that the answer to 30 x 20 is one hundred times bigger.

Multiplication should start with the ones of the second number. This ones digit should then be multiplied by the ones of the first number, then the tens and so on. Once the ones number has been multiplied then the tens/hundreds/thousands number should be multiplied in the same order.

More able children may be able to compact the algorithm like the example below:

$$
\begin{array}{rl}
 & 24 \\
\times & 38 \\
\hline
 & 192 \\
 & 7\overset{3}{\cancel{2}}0 \\
 & \overset{1}{\cancel{}} \\
\hline
 & 912 \\
 & \overset{1}{\cancel{}}
\end{array}
$$

When moving onto the tens multiplication (30 x 4 in the example to the left) you can explain to the children, who have secure place value understanding, that they can 'drop' a zero in the column before they multiply because numbers multiplied by ten always end in zero. Therefore, doing '3x4' would be the answer next to the zero. This should only be broached with children who have a very secure understanding of number and will often be taught in Year 6.

Multiplication

Y6

Write and calculate multiplication using multiplication tables, including for up to four-digit by two-digit numbers.

Children should be able to recall multiplication and division facts for multiplication tables up to 12 x 12. They continue to use partitioning to multiply basic two-digit times one-digit numbers and then use this knowledge of partitioning to transfer to a formal algorithm (see written methods section).

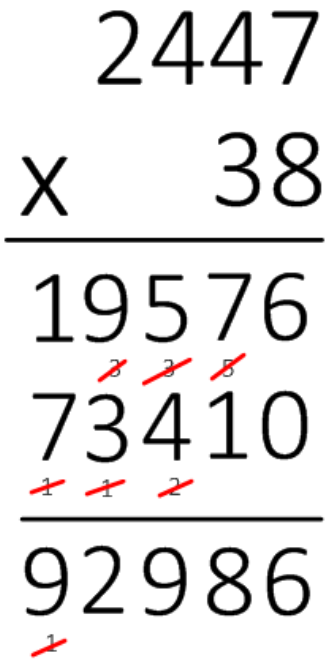
Mental methods

Identifying common factors and multiples of given numbers

Solving practical problems where children need to scale up. Relate to known number facts.

Written methods

Continue to refine and deepen understanding of written methods including fluency for using long multiplication. See below for an example of a compact method for calculating 4dx2d.



Division

Y3

Write and calculate division using multiplication tables, including for up two-digit numbers divided by 1 digit numbers with remainders.

Children should be able to use their understanding of multiplication and division facts of 3, 4 and 8 multiplication tables. They should know the difference between sharing and grouping to divide and understand how to use grouping on a numberline (see written methods section).

Mental methods

Halving 2 digit numbers using partitioning - making the link between fractions and division.

Children should be encouraged to use timestable facts to divide numbers that are within their timestable understanding.

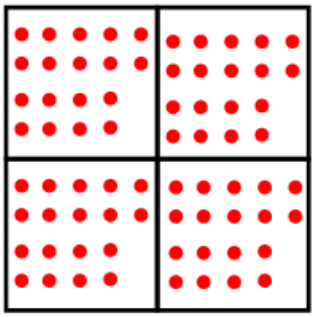
Written methods (progressing to 3d ÷ 1d)

Developing written methods using understanding of visual images. Use counters to physically share or group a number to find an answer (see below). When children are securing with grouping and sharing, demonstrate to children how grouping can be transferred to a numberline. Group horizontally initially, making it easy to transfer (although bear in mind that children see numberline horizontally and vertically).

Sharing

The 72 are *shared* between 4 parts.

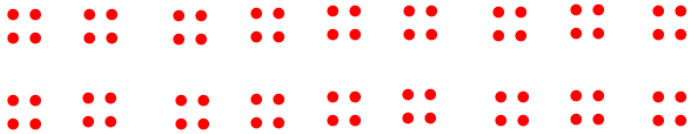
72 ÷ 4



Grouping

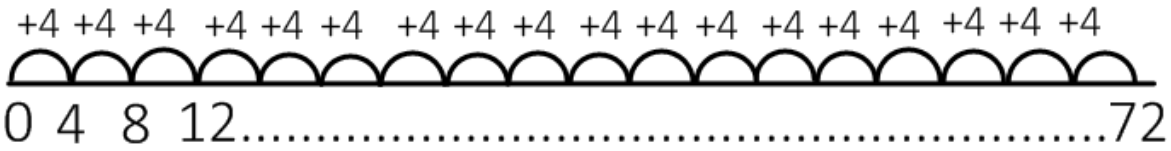
The 72 are *grouped* in 4s.

72 ÷ 4



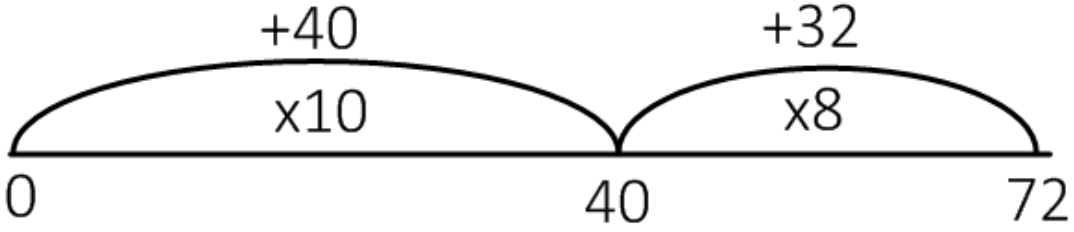
Grouping on a numberline

72 ÷ 4

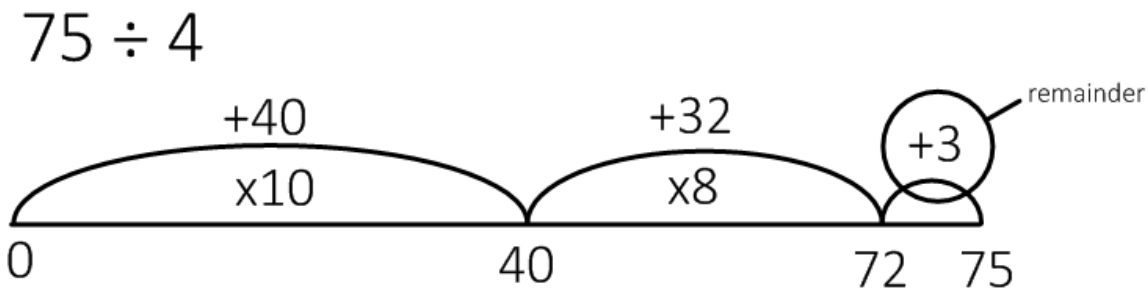


Eventually, children should be able to partition the dividend into numbers which aid the multiplication of the divisor.

72 ÷ 4



When children are secure with the method of grouping on a numberline, show them how remainders would look.



Division

Y4

Write and calculate division using multiplication tables, including for two-digit and three-digit numbers divided by up to two-digit numbers with remainders.

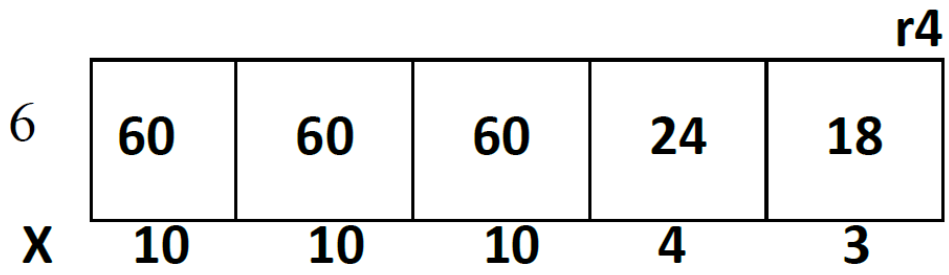
Children should be able to use their understanding of multiplication and division facts 12x12 multiplication tables. Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. Children should progress in their use of written division calculations.

Formal Written Methods

Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines in Year 3). Most Year 4 children will use the numberline method throughout Year 4. Very able students may be ready to move onto a more formal method (see Year 5).

The grouping on a numberline method could also be represented like the image below. This is a progression of the number line system as children no longer keep track of how close they are to the dividend, although they may make jottings.

166 ÷ 6 = 37



Division

Y5

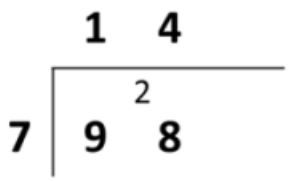
Write and calculate division using multiplication tables, including for up to four-digit numbers divided by one-digit numbers with remainders.

Children should be able to use their understanding of multiplication and division facts 12x12 multiplication tables. Children will be taught formal methods for calculating division, if they are ready. Short division ('bus stop') will be the main teaching point, with some children being taught long division later in the year.

Formal written methods

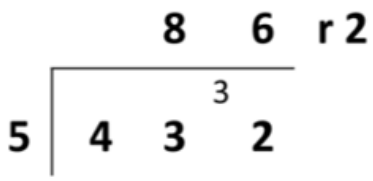
When ready, children will be taught the short division method ('bus stop'). Short division is the hardest method to link and explain place value. Therefore it is vital that children's place value understanding is very secure. See below for an example of the short division method.

98 ÷ 7 becomes



Answer: 14

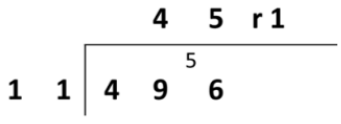
432 ÷ 5 becomes



Answer: 86 remainder 2

Children should begin to practically develop their understanding of how to express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1? How could I share this between 11 as well?)

496 ÷ 11 becomes



Answer: $45\frac{1}{11}$

Some children may be ready to move onto the long division method towards the end of the year.

432 ÷ 15 becomes

$$
\begin{array}{rr|lll}
 & & & 2 & 8 \\
\hline
1 & 5 & 4 & 3 & 2 \\
 & & 3 & 0 & 0 \quad 15\times20 \\
\hline
 & & 1 & 3 & 2 \\
 & & 1 & 2 & 0 \quad 15\times8 \\
\hline
 & & & 1 & 2
\end{array}
$$

$\cancel{12}/\cancel{15} = 4/5$

Answer: $28\frac{4}{5}$

Division

Y6

Write and calculate division using multiplication tables, including for up to four-digit numbers divided by two-digit numbers with remainders.

Children should be able to use their understanding of multiplication and division facts 12x12 multiplication tables. Children will continue to consolidate their understanding of short division. Some children may still be using grouping. Children will be taught the method of long division to tackle trickier questions.

Formal written methods

Children will use short division ('bus stop') method to solve questions which have one divisor. Children may be able to use the short division to solve questions with two divisors or they may use the long division method (see Year 5).

Children should always express remainders as a fraction or decimal. When ready, children may be taught to use short division to find a decimal answer. Ensure the link is made really explicit between fractions and division.

$\frac{12}{15} = 15\overline{)12}$

$$\begin{array}{r} 0.8 \\ 15\overline{)12.0} \end{array}$$

In the example on the left, children would hopefully realise that $\frac{12}{15}$ is equal to $\frac{4}{5}$ and therefore know it was 0.8.