PROGRESSION IN CALCULATION

This document has been created with reference to the Mathematics Mastery Progression document to provide clarity for staff.

It aims to provide guidance and ensure consistency in how children are taught to approach the four calculations without reliance on formal written methods from reception to year 6 at John of Gaunt Infant and Nursery School and Bure Valley School (Aylsham Learning Federation). Further, this document should give staff the confidence in using the core representations and manipulatives to provide children with the understanding of how to calculate. Progression guidance is not provided for nursery as the focus should be on the understanding of early number concepts and number sense through the use of concrete manipulatives.

Guidance has been arranged according to year group expectations, although it may be appropriate for teachers to make reference to lower year group guidance in order to meet the needs of all the children in the class. If children are deemed to be exceeding year group expectations, then they should not be moved onto higher year group content but should be guided to focus on the dimensions of depth.

This guidance should be used in conjunction with the Progression in Written Calculation document which outlines how to introduce children to formal methods (only relevant to year 3-6 Bure Valley School).

For the KS2 section, addition and subtraction have been combined as the key strategies overlap and the inverse of the operations is emphasised. Multiplication and division remain as individual sections.

It is important that children are equipped with the skills to solve problems they are presented with and they are able to apply these skills to real life contexts. As part of the teaching, children need to be taught how to independently select the best strategy for the calculation and understand how to support the structure of the mathematics with appropriate manipulatives.

Aims of the document

The Progression in Calculation document aims to:

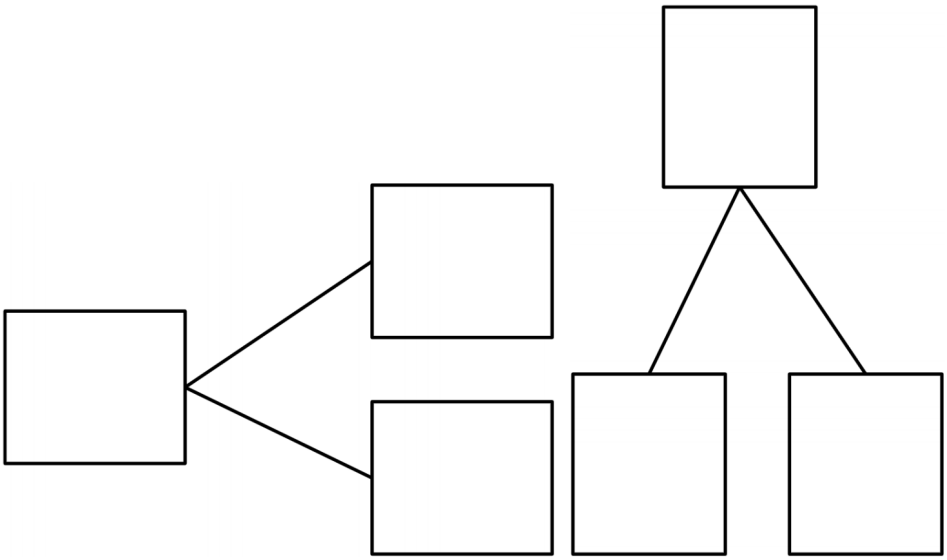
- Ensure all pupils have the potential to succeed when teaching using a mastery approach.
- Ensure pupils are equipped with the necessary calculation skills for their age.
- Develop teachers' understanding of how and when to introduce children to various representations to calculate with the four operations.
- Ensure that teachers know how to support children who may not yet be at their age related expectation .
- Equip pupils with the understanding of how to use particular representations and manipulatives to calculate without formal methods and to reveal structure.
- Promote the use of concrete manipulatives and pictorial representations for calculations and demonstrate how these can be used.
- Ensure children are able to explain their use of concrete manipulatives, pictorial and abstract representations, including any links made between them.
- Ensure a smooth transition between year groups, including from infant to junior phase.

Hierarchy of thinking

Can I do it in my head?
Can I use manipulatives to help me?
Can I use pictures or drawings to help me?
Which strategy will help me?

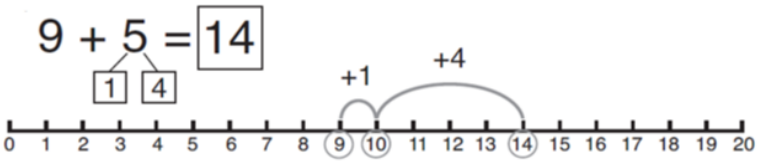
Core Representations

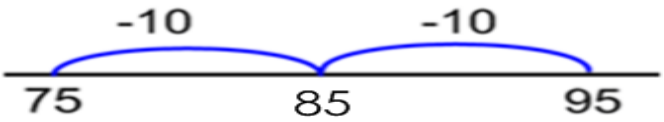
Part-whole Model



Tens Frame

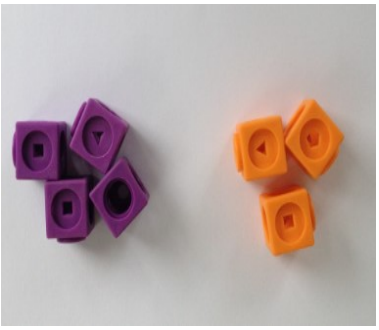
Number Lines





Core Manipulatives

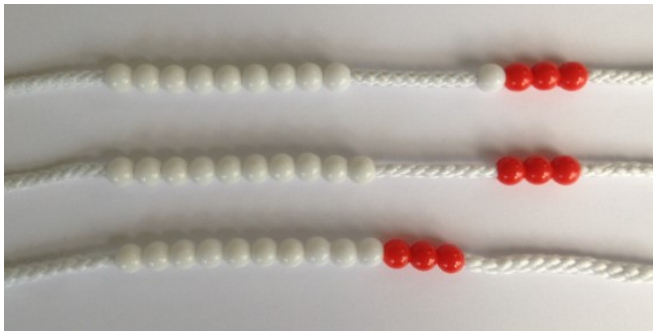
Cubes

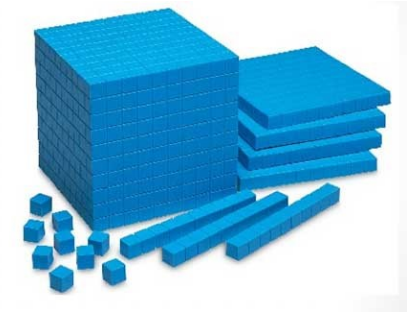
Counters

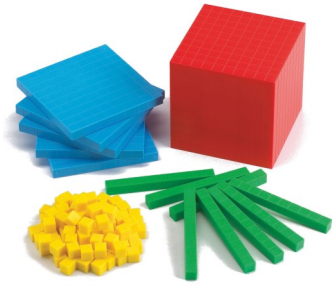


Bead Strings



Dienes





Cuisenaire Rods

Place Value Counters

Rationale

Reception – Early Years

In Reception the focus will be on the understanding of early number concepts and number sense. There will be an emphasis on using concrete manipulatives, songs and 'Big Pictures' to develop this understanding. Key vocabulary will be introduced. By the end of Reception children will have used tens frames and bead strings to support their addition and subtraction calculations. They will learn to use one-to-one correspondence, count all and counting on and back as strategies. They will be introduced to the concepts of grouping and sharing to develop their early understanding of multiplication and division. The continuous provision opportunities will allow them to explore maths and develop their understanding further.

Year1 - Lower KS1

When children start Year 1 they will have been understanding of early number concepts and number sense through the use of concrete manipulatives. They will have used concrete manipulatives, tens frames and bead strings to support their addition and subtraction calculations. They will have started to group and share concrete manipulatives for early multiplication and division calculations. In Year 1 they will build on the concrete understanding they have gained in Reception. They will learn to use part-whole, 'Make ten' and partitioning as strategies to support addition and subtraction calculations. They will learn how to use pictorial representations to support these calculations too and will start to write equations to show their methods. In multiplication, they will use skip counting in multiples of 2's, 5's and 10's and make equal groups with concrete manipulatives. They will also use the repeated addition strategy. In division they will use manipulatives to share objects into groups and how to represent this in a pictorial representation.

Year 2 - Upper KS1

When children start Year 2 they should already have an understanding of addition and subtraction operations. They will also have started to understand the basis of multiplication and division operations. They will continue to build on the concrete and conceptual understanding they have gained in Year 1 to develop their arithmetical confidence. In addition and subtraction, children will develop their understanding of the part-whole, 'Make ten' and partitioning strategies. They will be taught methods based on place value; this is a skill which supports both mental and written calculations. The use of larger numbers means that children need to be provided with the skills to move away from 'counting in ones' or using finger-based methods. A large focus in Year 2 is on multiples and near multiples of 10 and 100 where children are able to use complementary addition as an accurate means of achieving fast and accurate answers. Use of number lines and formal methods such as column addition and subtraction are introduced; initially children are not required to cross boundaries where they may 'carry', 'exchange' or r'egroup' to or from other columns. Column methods will also be presented in expanded forms to ensure place value is deep-rooted in their thinking. During this part of the key stage, multiplication and division facts are learnt, memorised and consolidated for 2, 3, 4, 5 and 10 times tables. Efficient pictorial methods are introduced for multiplying and dividing 2 digit numbers by a single-digit number, as are efficient mental strategies. For example, when multiplying or dividing by 4, children will be encouraged to double and double again, or half and half again, respectively; this relates the understanding of fractions and helps to secure this understanding. Similarly, when multiplying or dividing by numbers such as 5, children will be encouraged to build on their understanding of performing the same operation using 10.

Rationale

Year 3—LKS2

Year 3 pupils continue to build on the foundation set out for addition and subtraction in KS1. They will also continue to develop their multiplicative and distributive understanding. In addition and subtraction, children will learn to mentally add and subtract a three-digit number and ones, a three-digit number and tens and a three-digit number and hundreds. They will build on strategies such as round and adjust, derived number facts, 'make ten' and known facts within 20 to calculate mentally. They will use the bar model as a core representation as well as building on structures of the part-whole model and tens frames. On a more formal basis, children will learn to add and subtract up to 4 digit numbers using formal written methods. Pupils will continue to develop their multiplication and distributive understanding by counting from 0 in multiples of 4, 8, 50 and 100; they will also build on their known multiplication and division facts by learning 3, 4, 6 and 8 multiplication tables. Children will use mental methods to calculate one-digit times two-digit that are multiplied and divided. In addition to this, children will learn how to partition the number to multiply the two-digit number by a one-digit number. This will build on the partitioning work children have done in KS1. Ten times greater is a core strategy for year 3 as they are able to build on the work with dienes from KS1. Dienes equipment should continue to be used as a representation to deepen the understanding of 10 times greater or smaller. Children will use known facts for multiplying by 10 and 100. This will build on the 10 times greater concept. For division, year 3 will only build on their division skills from year 2 by introducing divide by the powers of 10. They'll use the strategy of 'scaling down' to carry out the method.

Year 4—LKS2

Pupils will build on their formal method work from year 3 adding and subtracting 4 digit numbers using the column method. They'll also begin to find 1,000 more or less than a given number. Children will use known facts and knowledge of place value to derive facts. Place value counters will be used to represent the connection between known facts and place value. They are a core representation for year 4. The focus for addition and subtraction shifts to decimals in year 4 and children will use Dienes and place value counters to represent the decimals. The methodology remains the same but the value of the mathematical equipment is now altered. They should also make reference to a number bead as a representation for decimals, building on the work they did in KS1. For multiplication and division, children move on to counting in multiples of 6, 7, 9, 25 and 1000. The focus for representation should be arrays extended to use multiples of 10 place value counters. They should also use the array to partition one number and multiply each part. Children can then use the same method (array with multiples of 10 counters) to group and solve division problems. The short division method should also make use of multiples of 10 counters.

Year 5&6—UKS2

Year 5&6, for the purpose of this document, have been combined. This is because the calculation strategies used are broadly similar, with year 6 using larger and smaller (decimals) numbers. Both year groups will begin working with numbers that increase in size in comparison to earlier years. Year 5 should work with numbers up to 7 digits and year 6 should work with numbers up to 8 digits. They should focus on methods which draw on their understanding of numbers– e.g. using factors and the distributive and associative laws. The focus on strategies should be on reinforcing skills such as partitioning to calculate and using efficient methods such as doubling and halving to find answers. In addition to this, work should continue around numbers being 10, 100, 100 times larger and smaller to calculate an answer. Children should use the number line to reinforce finding the difference in connection with subtraction. No reference is made to any formal methods in this document as the focus is on alternative strategies. An emphasis should be made that formal written methods are not always the most efficient choice to solve a calculation. A separate document *'Progression in written calculation methods'* should be used for guidance around teaching formal written methods.

Addition

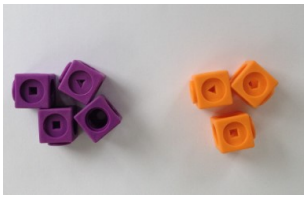
R

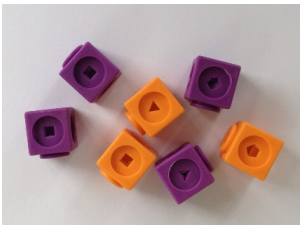
Adding up to 20

Count all

Joining two groups and then recounting all objects using one-to-one correspondence.

3 + 4 = 7





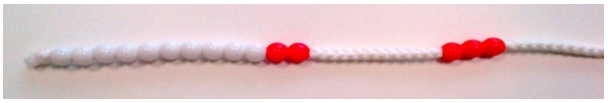
5 + 3 = 8

Counting on

As a strategy, this should be limited to adding small quantities only (1, 2 or 3) with pupils understanding that counting on from the greater number is more efficient.

8 + 1 = 9

15 = 12 + 3



Addition

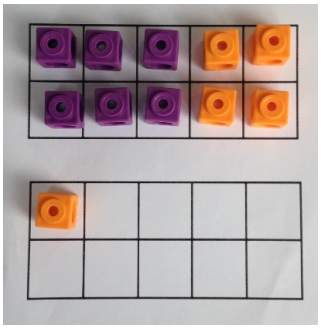
R

Adding up to 20

Using Tens frames to support bridging 10

As a starting point to the 'Make ten' strategy that is taught in Year 1, the pupils will use tens frames to understand that when adding two groups we sometimes need to bridge 10.

6 + 5 = 11



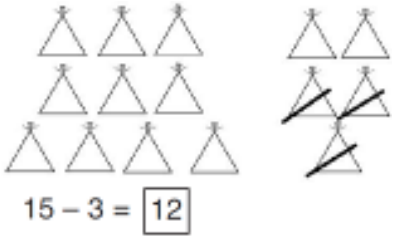
Subtraction

R

Subtracting numbers under 20

Taking away from the ones

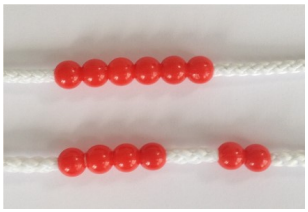
When this is first introduced, the concrete representation should used so the pupils have a set of objects and take away part of that set.
Then the concrete representation should be based upon the diagram. Real objects should be placed on top of the images as one-to-one correspondence so that pupils can take them away.



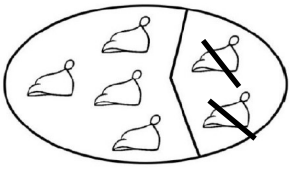
Counting back

Subtracting 1, 2 or 3 by counting back.
Pupils should be encouraged to rely on number bonds knowledge as time goes on, rather than using counting back as their main strategy.

4 = 6 - 2



16 - 2 = 14



Subtraction

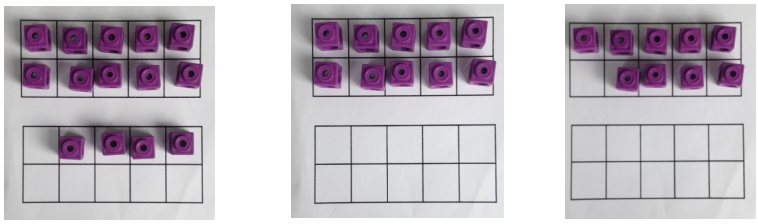
R

Subtracting numbers under 20

Using Tens frames to support bridging 10

As a starting point to the 'Make ten' strategy that is taught in Year 1, the pupils will use tens frames to understand that when subtracting two groups we sometimes need to bridge 10.

14 - 5 = 9



Multiplication

R

Doubling

Skip counting in multiples of 1 and 2 from zero

The representation for the amount of groups supports pupils' understanding of the written equation. So two groups of 2 are 2, 4. Or five groups of 2 are 2, 4, 6, 8, 10.
Count the groups as pupils are skip counting.
Number lines can be used in the same way as the bead string.
Pupils can use their fingers as they are skip counting.

Making 2 equal groups and counting the total - Doubling

The importance should be placed on the vocabulary used.

Division

R

Sharing objects into 2 groups

Pupils should become familiar with halving through working practically. The division symbol is not taught at this stage.

Addition

Y1

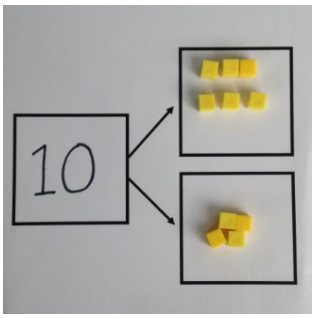
Adding up to two 2-digit numbers

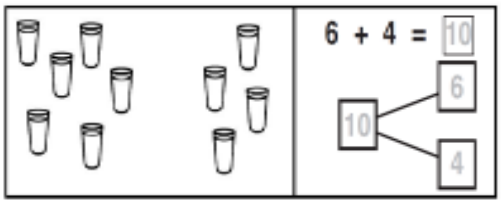
Part-whole

Pupils explore the different ways of making 10 then 20. They can do this with all numbers using the same representations.

We teach both addition and subtraction alongside each other, as pupils will use this model to identify the inverse relationship between them.

This model begins to develop the understanding of the commutativity of addition, as pupils become aware that the parts will make the whole in any order.





10 = 6 + 4
10 – 6 = 4
10 - 4 = 6
10 = 4 + 6

Addition

Y1

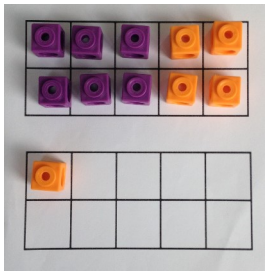
Adding up to two 2-digit numbers

'Make ten' strategy

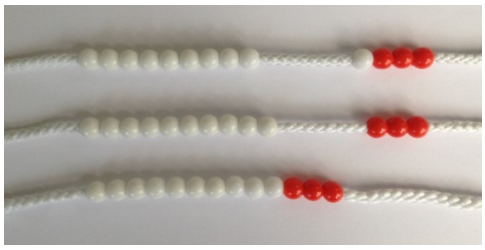
Pupils should be encouraged to start at the greater number and partition the smaller number to make ten.

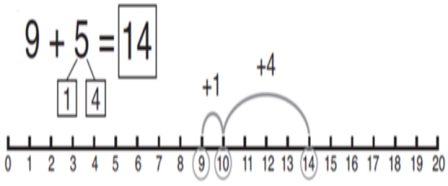
The colours of the beads on the bead string make it clear how many more need to be added to make ten.

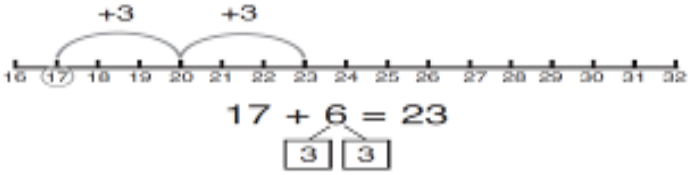
6 + 5 = 11



9 + 4 = 13







Addition

Y1

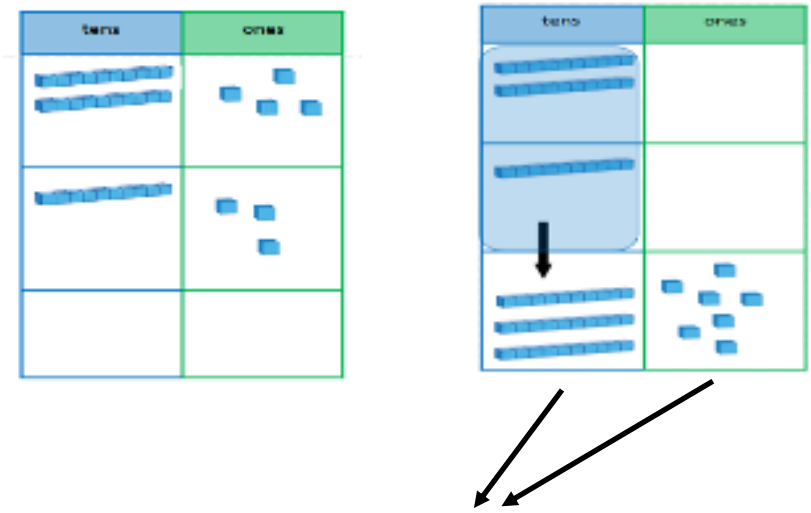
Adding up to 2 digit numbers

Partitioning to add (no regrouping)

Place value grids and Dienes blocks could be used as shown in the diagram before moving onto pictorial representations. Dienes blocks should always be available, as the main focus in Year 1 is the concept of place value rather than mastering the procedure.

When not regrouping, partitioning is a mental strategy and does not need formal recording in columns. This representation prepares them for using column addition with formal recording.

24 + 13 = 37



24 + 13 = 37

Subtraction

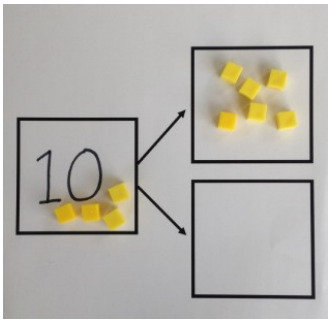
Y1

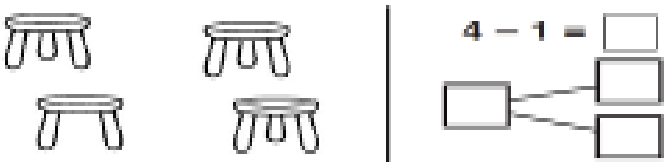
Subtracting numbers up to 2 digits

Part-whole

Teach both addition and subtraction alongside each other, as the pupils will use this model to identify the link between them.
Pupils start with ten cubes placed on the whole.
They then remove what is being taken away from the whole and place it on one of the parts. The remaining cubes are the other part and also the answer. These can be moved into the second part space.

10 - 6 = 4

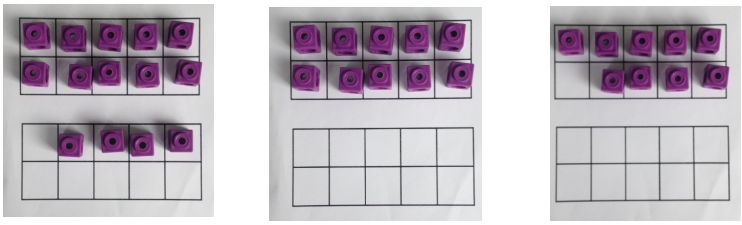


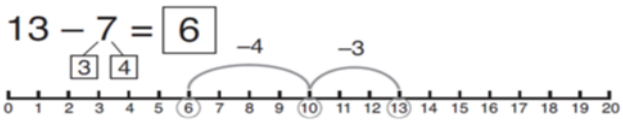


'Make ten' strategy

To subtract a 1-digit number from a 2-digit number.
Pupils identify how many need to be taken away to make ten first, partitioning the number being subtracted. Then they take away the rest to reach the answer.

14 - 5 = 9





Subtraction

Y1

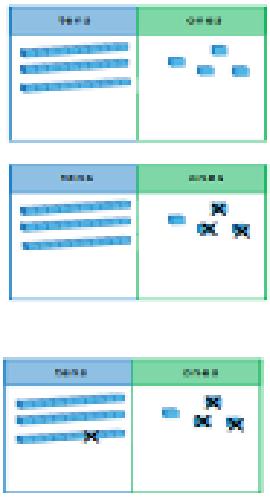
Subtracting numbers up to 2 digits

Partitioning to subtract without regrouping

Dienes blocks on a place value chart (developing into using images on the chart) could be used, as when adding 2-digit numbers, reinforcing the main concept of place value for Year 1.

When not regrouping, partitioning is a mental strategy and does not need formal recording in columns. This representation prepares them for using column subtraction with formal recording.

34 - 13 = 21



34 - 13 = 21

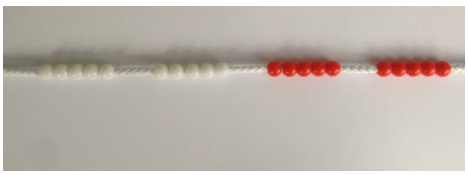
Multiplication

Y1

Solving one-step problems using multiples of 2, 5 and 10.

Skip counting in multiples of 2, 5, 10 from zero

The representation for the amount of groups supports pupils' understanding of the written equation. So two groups of 2 are 2, 4. Or five groups of 2 are 2, 4, 6, 8, 10.
Count the groups as pupils are skip counting.
Number lines can be used in the same way as the bead string.
Pupils can use their fingers as they are skip counting.

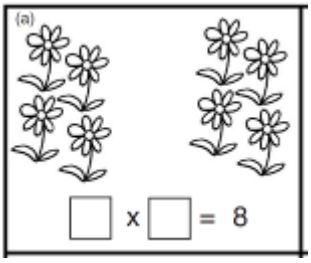


4 x 5 = 20

2 x 4 = 8

Making equal groups and counting the total

How this would be represented as an equation will vary. This could be 2 × 4 or 4 × 2.
The importance should be placed on the vocabulary used alongside the equation. So this picture could represent 2 groups of 4 or 4 twice.



Draw  to show 2 x 3 = 6

Multiplication

Y1

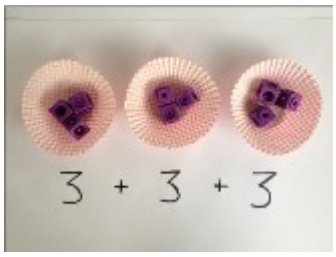
Solving one-step problems using multiples of 2, 5 and 10.

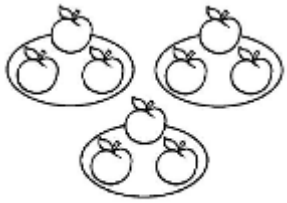
Solve multiplications using repeated addition

This strategy helps pupils make a clear link between multiplication and division as well as exemplifying the 'repeated addition' structure for multiplication.

It is a natural progression from the previous 'count all' strategy as pupils can be encouraged to 'count on'. However, as number bonds knowledge grows, pupils should rely more on these important facts to calculate efficiently.

3 x 3 = 3 + 3 + 3





How many apples are there altogether?

3 + 3 + 3 = 9

Division

Solving one-step problems using multiples of 2, 5 and 10.

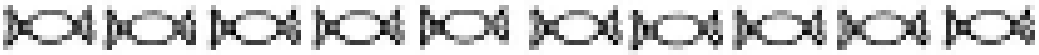
Y1

Sharing objects into groups

Pupils should become familiar with division equations through working practically. The division symbol is not formally taught at this stage.

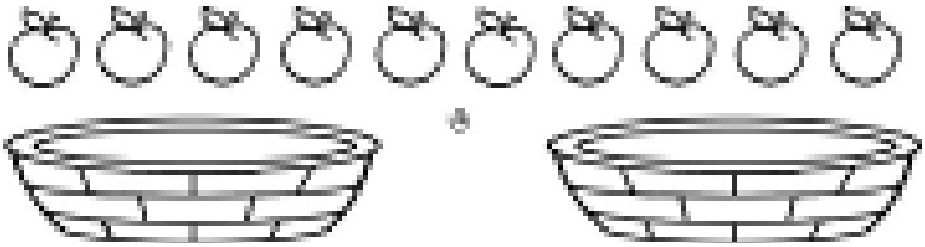
10 ÷ 2 = 5

There are 10 sweets. Ring groups of 2.



There are _____ groups of 2.

Draw an equal number of apples for each basket.



There are five apples in each basket.

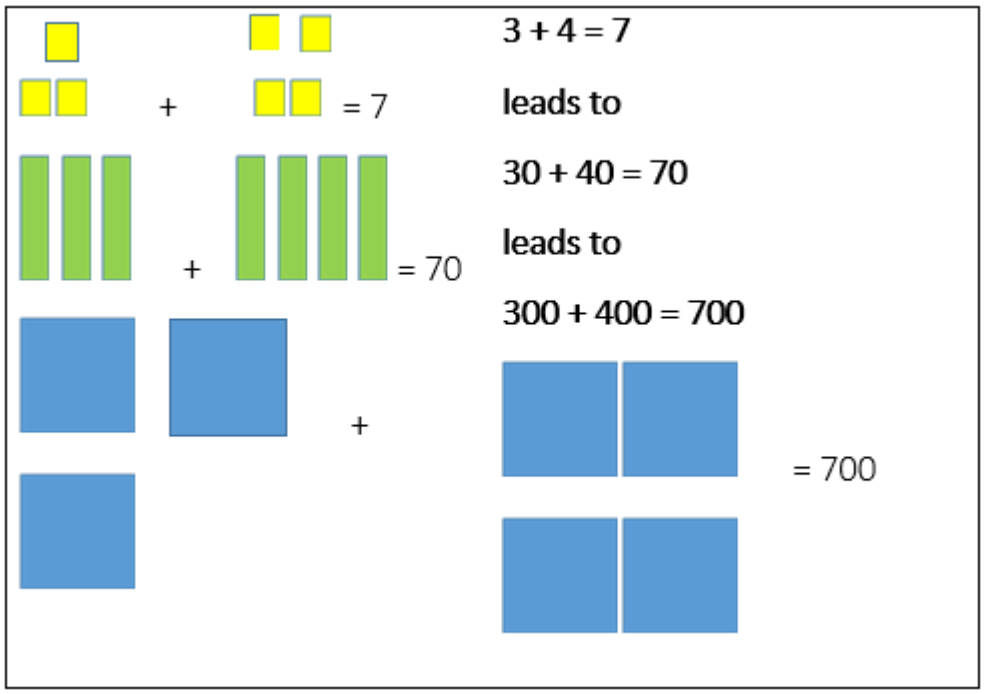
Addition

Adding 2 & 3-digits numbers

Y2

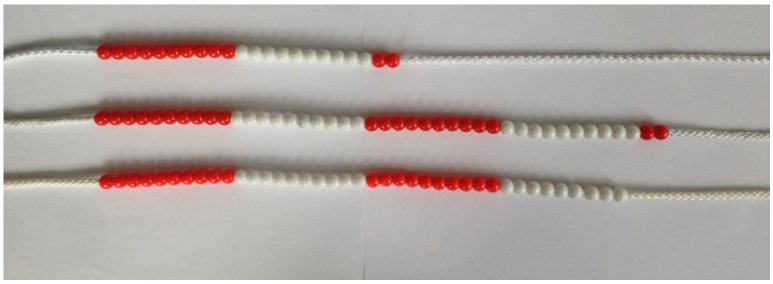
Using known facts to create derived facts

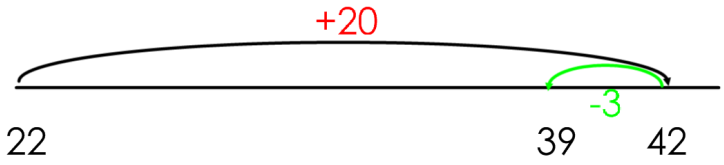
Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy.



Round and adjust (sometimes known as a compensating strategy)

Pupils will develop a sense of efficiency with this method, beginning to see when rounding and adjusting is more efficient than adding tens and then ones.





22 + 17 = 39

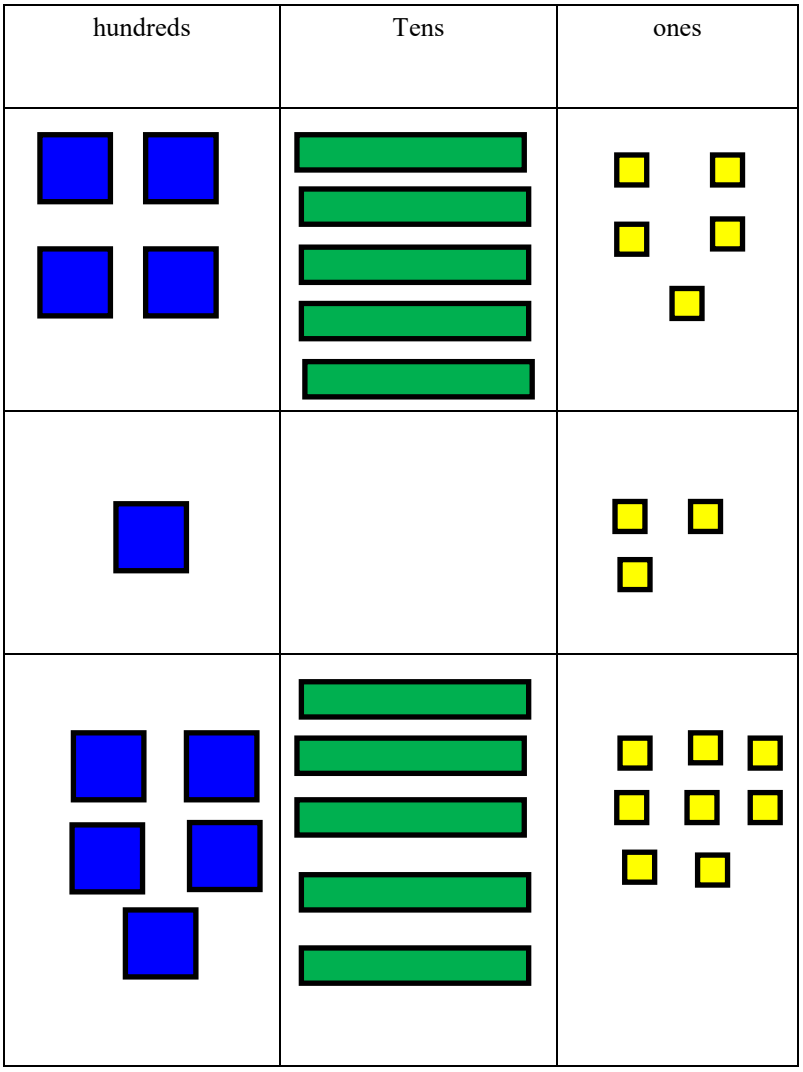
Addition

Y2

Adding 2 & 3-digits numbers

Partitioning to add without regrouping

As in Year 1, this is a mental strategy rather than a formal written method. Pupils use the Dienes blocks (and later, images) to represent 3- digit numbers but do not record a formal written method if there is no regrouping.



4 5 5 + 1 0 3 = 5 5 8

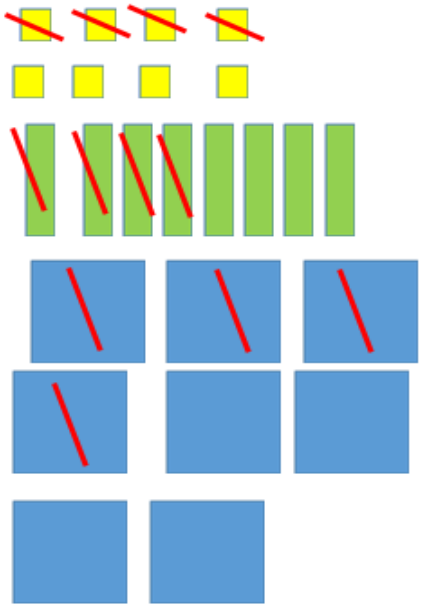
Subtraction

Y2

Subtracting 2 & 3-digit numbers

Using known number facts to create derived facts

Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy, encouraging pupils to apply their knowledge of number bonds to subtract multiples of 10 and 100.



8 - 4 = 4

leads to

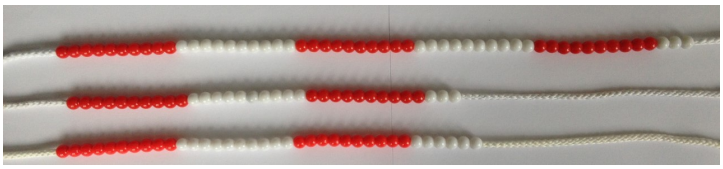
80 - 40 = 40

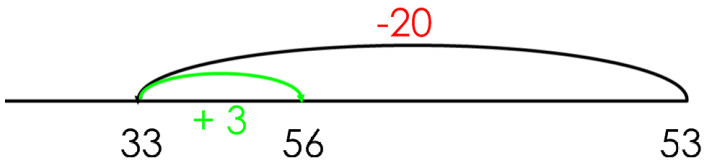
leads to

800 - 400 = 400

Round and adjust (sometimes known as a compensating strategy)

Pupils must be taught to round the number that is being subtracted.
Pupils will develop a sense of efficiency with this method, beginning to identify when this method is more efficient than subtracting tens and ones.





53 - 17 = 36

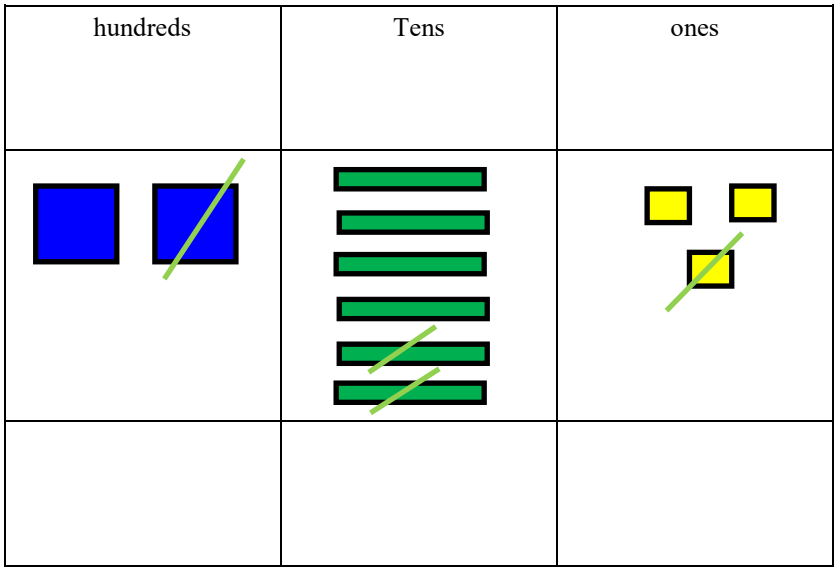
Subtraction

Y2

Subtracting 2 & 3-digit numbers

Partitioning to subtract without regrouping

As in Year 1, the focus is to develop a strong understanding of place value and pupils should always be using concrete manipulatives alongside the pictorial. Formal recording in columns is unnecessary for this mental strategy. It prepares them to subtract with 3-digits when regrouping is required.



263 – 121= 142

Multiplication

Y2

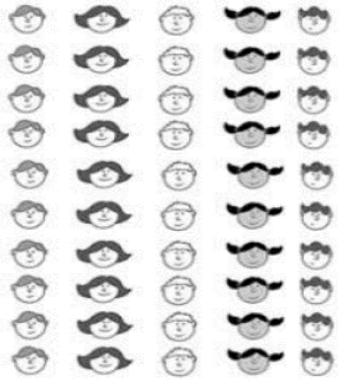
Solving simple problems using the 2, 5 and 10 times tables.

Arrays to represent multiplication equations

Concrete manipulatives and images of familiar objects begin to be organised into arrays and, later, are shown alongside dot arrays. It is important to discuss with pupils how arrays can be useful.

Pupils begin to understand multiplication in a more abstract fashion, applying their skip counting skills to identify the multiples of the 2x, 5x and 10x tables.

The relationship between multiplication and division also begins to be demonstrated.



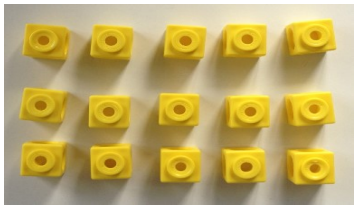
Multiplication

Y2

Solving simple problems using the 2, 5 and 10 times tables.

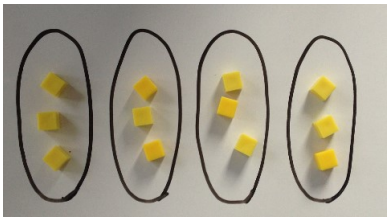
Multiplication is commutative

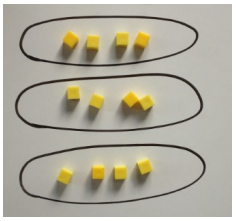
Pupils should understand that an array and, later, bar models can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.

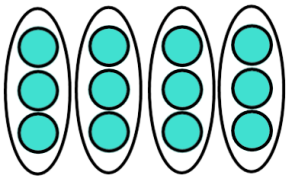


3 x 5 = ☐

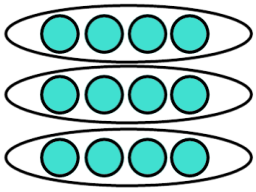
5 x 3 = ☐







12 = 3 × 4



12 = 4 × 3

Multiplication

Y2

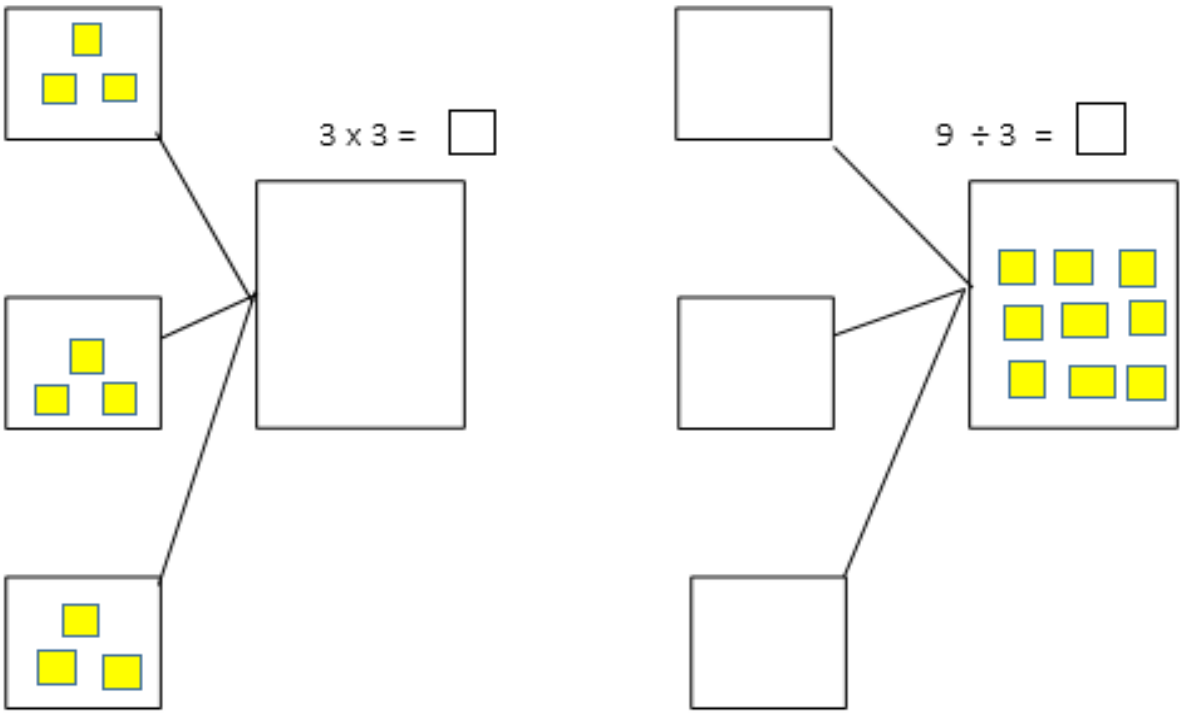
Solving simple problems using the 2, 5 and 10 times tables.

Use of part-whole model to establish the inverse relationship between multiplication and division

This link should be made explicit from early on, using the language of the part-whole model, so that pupils develop an early understanding of the relationship between multiplication and division. Bar models (with Cuisenaire rods) should be used to identify the whole, the value of the parts and the number of parts.

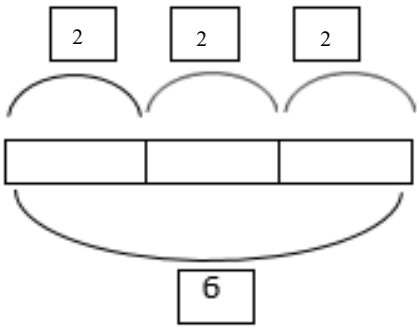
It is important to highlight that with multiplication, the parts are of equal value as this is different to how this model is used for addition and subtraction.

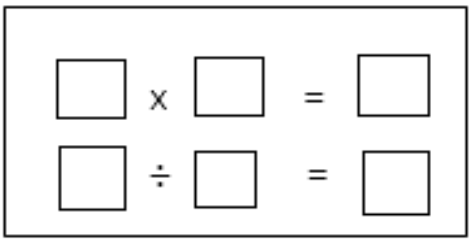
There are three equal parts. Each part has a value of three. What is the whole?



What multiplication and division equations can you write for each bar model?

Prove that the equations are correct using a bead string.





Division

Y2

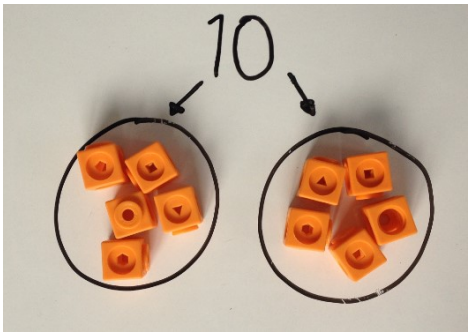
Solving simple problems using the 2, 5 and 10 times tables.

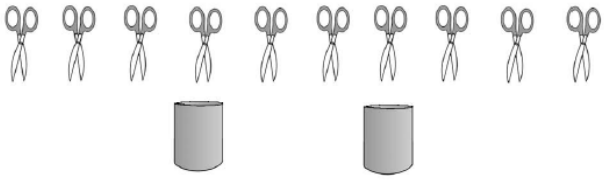
Division as sharing

Here, division is shown as sharing.

If we have ten pairs of scissors and we share them between two pots, there will be 5 pairs of scissors in each pot.

10 ÷ 2 = 5



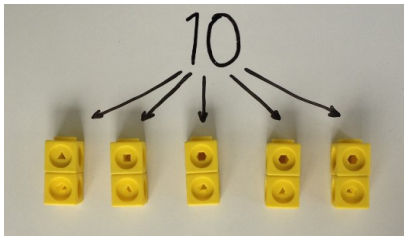


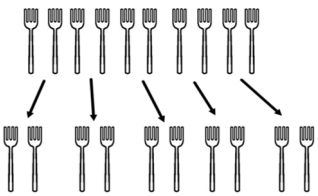
Division as grouping

Here, division is shown as grouping.

If we have ten forks and we put them into groups of two, there are 5 groups.

10 ÷ 2 = 5





Division

Solving simple problems using the 2, 5 and 10 times tables.

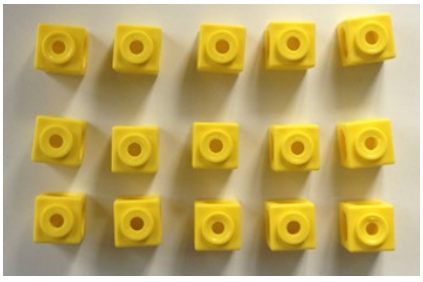
Y2

Use of part-whole model to represent division equations and to emphasise the relationship between division and multiplication

Pupils use arrays of concrete manipulatives and images of familiar objects to solve division equations.

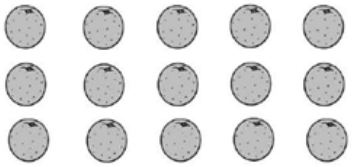
They begin to use dot arrays to develop a more abstract concept of division.

It is important to highlight that with multiplication and division, the parts are of equal value as this is different to how this model is used for addition and subtraction.

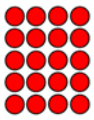


15 ÷ 5 = 3

15 ÷ 3 = 5

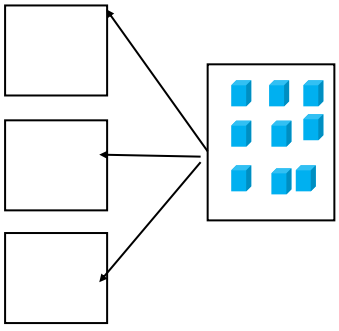


Write the division equations that this array represents.



20 ÷ 4 = ☐ 20 ÷ 5 = ☐

The whole is nine. There are three equal parts. What is the value of each part?



Addition & Subtraction Y3

Adding 3 & 4-digit numbers

Add and subtract numbers mentally

Pupils learn that this is an appropriate strategy when they are able to use known and derived number facts or other mental strategies to complete mental calculations with accuracy.

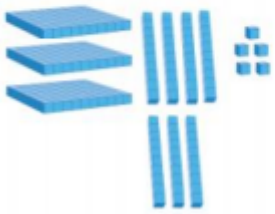
To begin with, some pupils will prefer to use this strategy only when there is no need to regroup, using number facts within 10 and derivations. More confident pupils might choose from a range of mental strategies that avoid written algorithms, including (but not exhaustively):

- known number facts within 20
- derived number facts,
- 'Make ten',
- round and adjust

It is important to model the mental strategy using concrete manipulatives in the first instance.

No Regrouping

345 + 30	274 - 50
1128 + 300	1312 - 300
326 + 342	856 - 724



I know 4 + 3 = 7, so 4 tens plus 3 tens is equal to 7 tens.
345 + 30 = 375.

With some regrouping

416 + 25	232 - 5
383 + 130	455 - 216
611 + 194	130 - 40
1482 + 900	2382 - 500

Deriving facts

Pupils use known facts such as number bonds and understanding of place value and magnitude to derive further facts.

If I know $12 + 5 = 17$ *then* $22 + 5 = 27$.
If I know $12 + 5 = 17$ *then* $17 - 12 = 5$
If I know $17 - 12 = 5$ *then* $37 - 12 = 25$

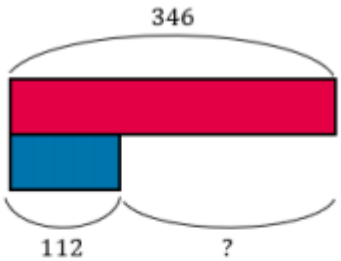
Addition & Subtraction

Adding up to 3 & 4-digit numbers

Y3

Bar models

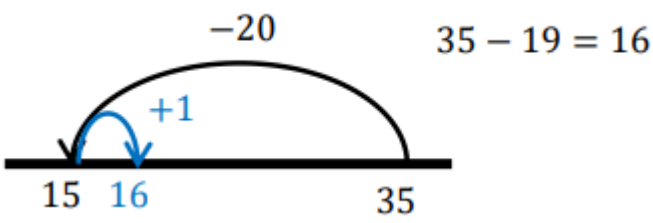
Pictorial bar models and concrete Cuisenaire as bar models are used throughout the year and represent part-whole relationships and knowns and unknowns within problems.



I know the whole is 346, and one of the parts is 112. I do not know the value of the missing part. I can subtract 112 from 346.

Round and adjust

Pupils apply understanding of ordinality of number, recognising when a part or whole is close to a multiple of 10 e.g. 29, 32. They round before calculating, then adjust their answer accordingly. Concrete or pictorial models are used to represent this.



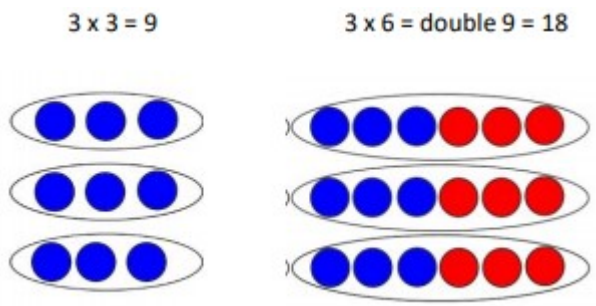
Multiplication

Y3

Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers.

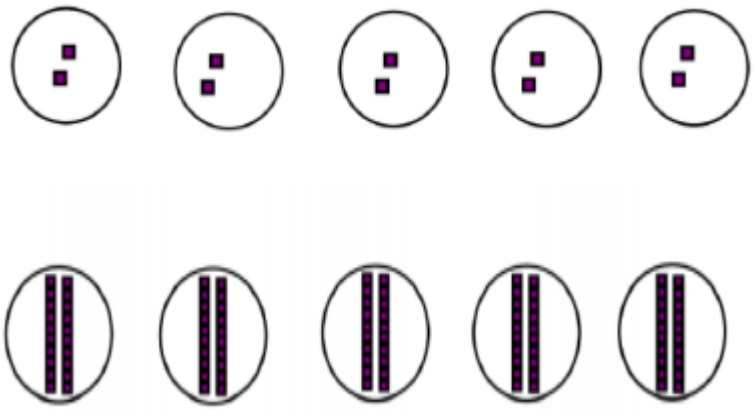
Doubling to derive new multiplication facts

Pupils continue to make use of the idea that facts from easier times tables can be used to derive facts from related times tables using doubling as a strategy. This builds on the doubling strategy from Year 2.



Ten times greater

Pupils' work on this must be firmly based on concrete representations – the language of ten times greater must be well modelled and understood to prevent the numerical misconception of 'adding a zero'.



Multiplication

Y3

Solving simple problems using the 2, 5 and 10 times tables.

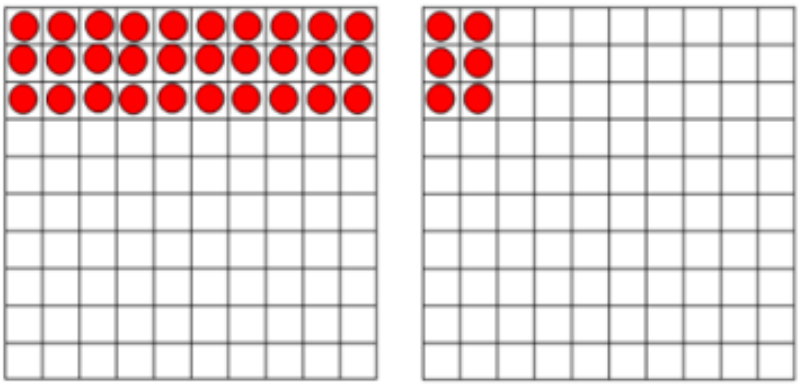
Multiplication of 2- digit numbers with partitioning (no regrouping)

Children should always consider whether partitioning is the best strategy – if it is possible to use strategies such as doubling (some may use doubling twice for ×4), they need to choose the most efficient strategy. Children may wish to make jottings, including a full grid as exemplified here – but grid method is not a formal method and its only purpose is to record mental calculations. This supports the development of the necessary mental calculating skills but does not hinder the introduction of formal written methods in Year 4. Concrete manipulatives are essential to develop understanding.

3 x 12

12 = 10 + 2

3 x 10 | 3 x 2



Now add the total number of tens and ones

×	10	2
3		

×	10	2
3	30	6

3 x 12 = 36

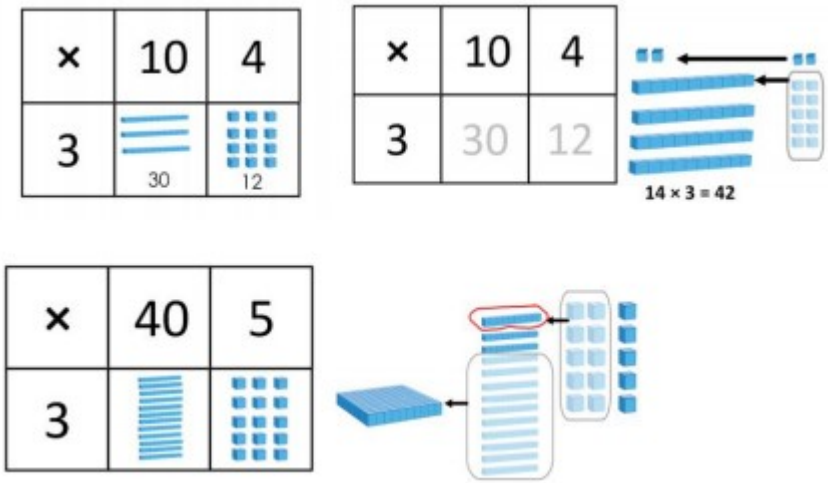
Multiplication

Y3

Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers.

Multiplication of 2- digit numbers with partitioning (regrouping)

Using concrete manipulatives and later moving to using images that represent them, supports pupils' early understanding, leading towards formal written methods in Year 4. Once again, this is a mental strategy, which they may choose to support with informal jottings, including a full grid, as exemplified here. Pupils must be encouraged to make use of their known multiplication facts and their knowledge of place value to calculate, rather than counting manipulatives.



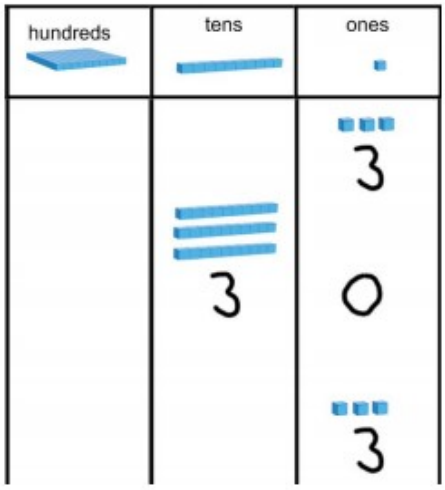
Division

Y3

Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers.

Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using scaling down

Pupils use the strategy of 'scaling down', representing numbers with concrete manipulatives and making the value ten times smaller.



3 × 10 = 30

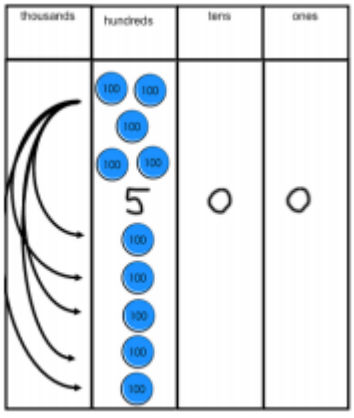
30 ÷ 10 = 3

Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using grouping

Pupils divide by 10, 100 and 1000 by making groups of the divisor.

500 ÷ 100 = ☐

My whole is 500 and the value of the equal parts is 100. How many parts are there?

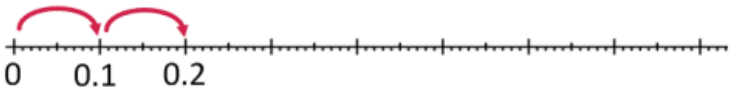


Addition & Subtraction Y4

Add numbers up to 4-digit numbers

Count forwards and backwards in steps of 10, 100 and 1000 for any number up to 10 000.

Pupils should count on and back in steps of ten, one hundred and one thousand from different starting points. These should be practised regularly, ensuring that boundaries where more than one digit changes are included.



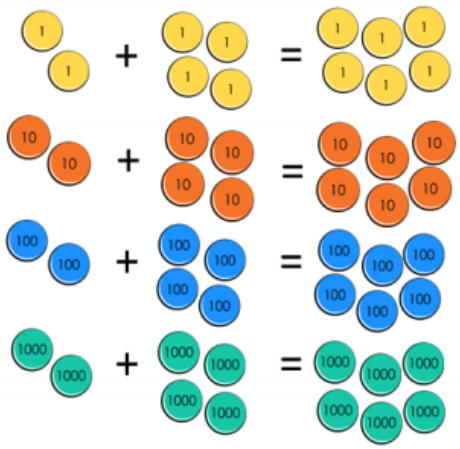
Pay particular attention to boundaries where regrouping happens more than once and so more than one digit changes.

E.g. 990 + 10 or 19.9 + 0.1

Count forwards and backwards in tenths and hundredths.

Using known facts and knowledge of place value to derive facts. Add and subtract multiples of 10, 100 and 1000 mentally

Pupils extend this knowledge to mentally adding and subtracting multiples of 10, 100 and 1000. Counting in different multiples of 10, 100 and 1000 should be incorporated into transition activities and practised regularly.



2 + 4 = 6

20 + 40 = 60

200 + 400 = 600

2000 + 4000 = 6000

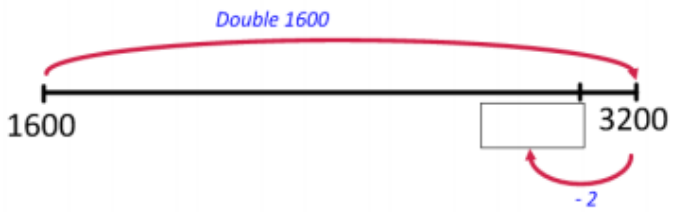
Addition & Subtraction Y4

Add numbers up to 4-digits numbers

Near doubles

Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten. These facts can be adjusted to calculate near doubles.

1600 + 1598 = double 1600 – 2



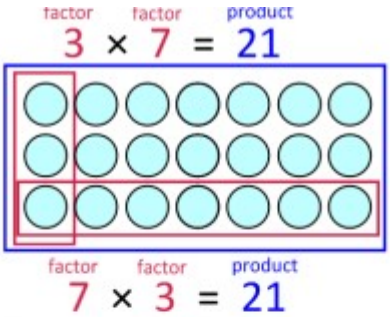
Multiplication

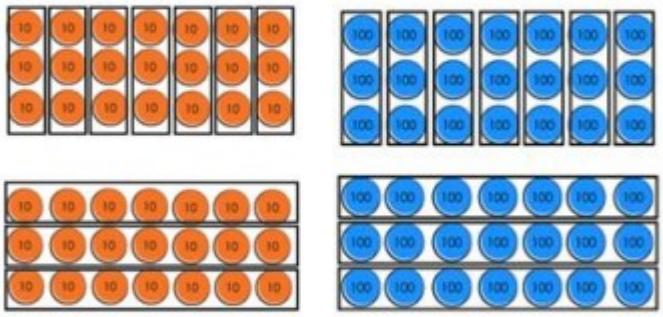
Y4

Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers.

Using known facts and place value for mental multiplication involving multiples of 10 and 100

Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally. Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact. Knowledge of commutativity (that multiplication can be completed in any order) is used to find a range of related facts.





30 x 7 = 210	300 x 7 = 2100
70 x 3 = 210	700 x 3 = 2100
7 x 30 = 210	7 x 300 = 2100
3 x 70 = 210	3 x 700 = 2100

Multiplication

Y4

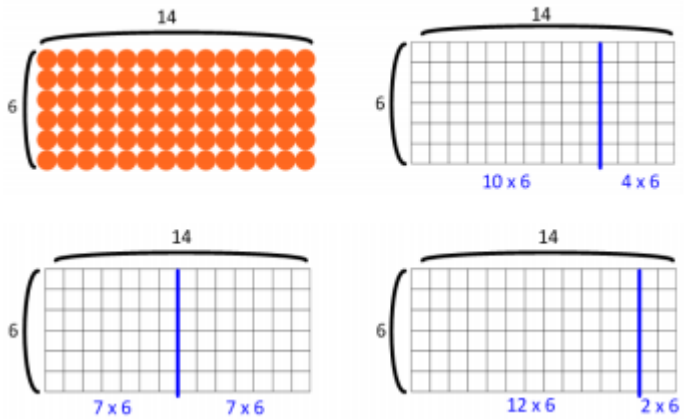
Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers.

Multiplying by partitioning one number and multiplying each part

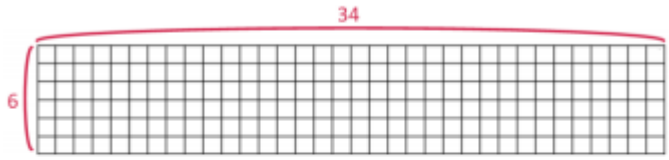
Pupils build on mental multiplication strategies and develop an explicit understanding of distributive law, which allows them to explore new strategies to make more efficient calculations. As well as partitioning into tens and ones (a familiar strategy), they begin to explore compensating strategies and factorisation to find the most efficient solution to a calculation.

Distributive law a x (b + c) = a x b + a x c

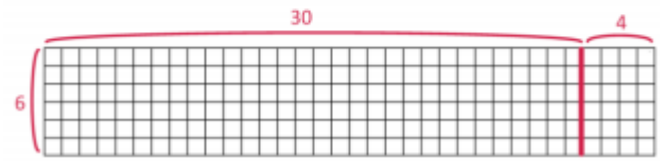
14 x 6



34 × 6



30 × 6 + 4 × 6



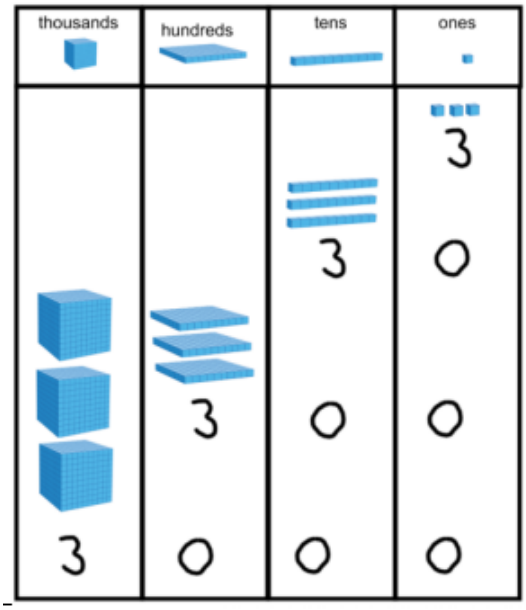
Division

Y4

Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers.

Dividing by 10 and 100

When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller. When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller. E.g. 210 ÷ 10 = 21.



30 ÷ 10 = 3
300 ÷ 100 = 3
3000 ÷ 1000 = 3

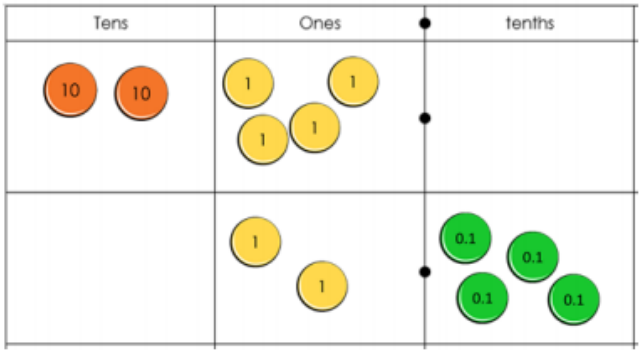
300 ÷ 10 = 30
3000 ÷ 100 = 30

3000 ÷ 10 = 300

Division of a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths

When you divide by ten, each part is ten times smaller. The tens become ones and the ones become tenths. Each digit is in a place that gives it a value that is ten times smaller.

24 ÷ 10 = 2.4



Division

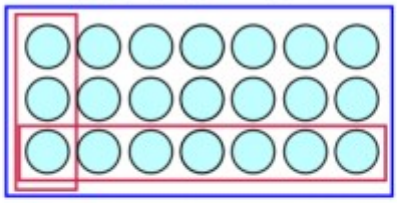
Y4

Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers.

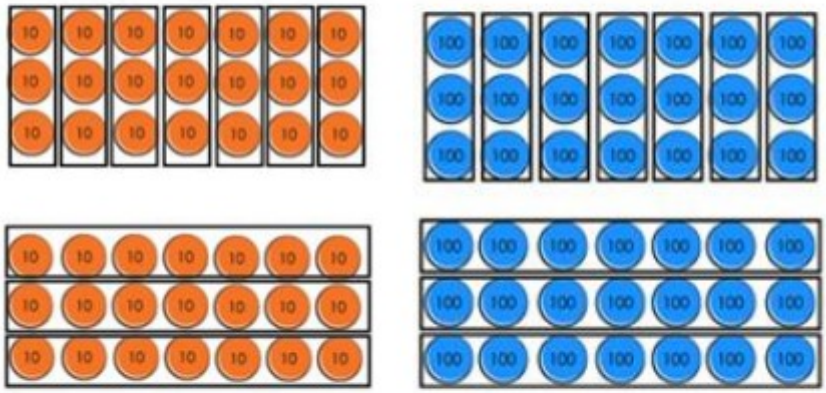
Derived facts

Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally. Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.

21 ÷ 3 = 7



21 ÷ 7 = 3



210 ÷ 7 = 30
210 ÷ 3 = 70
210 ÷ 30 = 7
210 ÷ 70 = 3

2100 ÷ 7 = 300
2100 ÷ 3 = 700
2100 ÷ 300 = 7
2100 ÷ 700 = 3

Y5 Addition & Subtraction Y6

Add and subtract numbers mentally with increasingly large numbers

Partitioning one number and applying known facts to add.

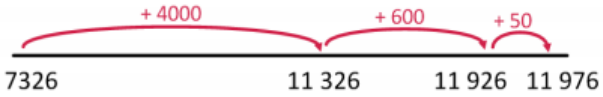
Pupils can use this strategy mentally or with jottings as needed. Pupils should be aware of the range of choices available when deciding how to partition the number that is to be added. They should be encouraged to count on from the number of greater value as this will be more efficient. However, they should have an understanding of the commutative law of addition, that the parts can be added in any order. Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.

Partitioning into place value amounts (canonical partitioning):

4650 + 7326 = 7326 + 4000 + 600 + 50

With place value counters, represent the larger number and then add each place value part of the other number. The image above shows the thousands being added.

Represent pictorially with an empty numberline:



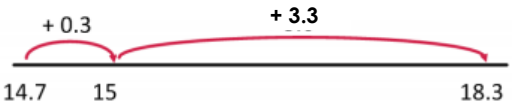
Partitioning in different ways (non-canonical partitioning):

Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count on to a multiple of 10.

6785 + 2325 = 6785 + 15 + 200 + 2110

The strategy can be used with decimal numbers, Make one:

14.7 + 3.6 = 14.7 + 0.3 + 3.3 = 15 + 3.3



Y5 Addition & Subtraction Y6

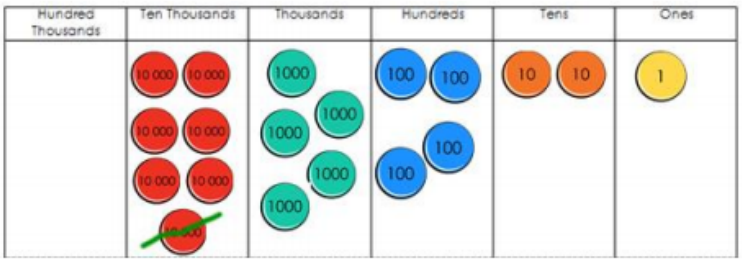
Add and subtract numbers mentally with increasingly large numbers

Subtraction by partitioning and applying known facts.

Pupils can use this strategy mentally or with jottings as needed. Pupils should be aware of the range of choices available when deciding how to partition the number that is to be subtracted. Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.

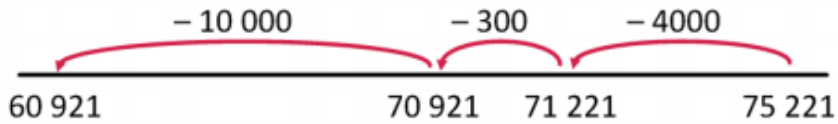
Partitioning into place value amounts (canonical partitioning):

75 221 – 14 300 = 75 221 – 10 000 – 4000 – 300



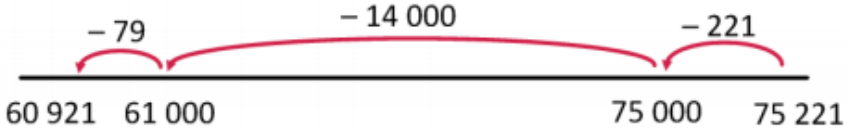
Represent pictorially with a number line, starting on the right and having the arrows jump to the left:

Develop understanding that the parts can be subtracted in any order and the result will be the same:



Partitioning in different ways (non-canonical partitioning):

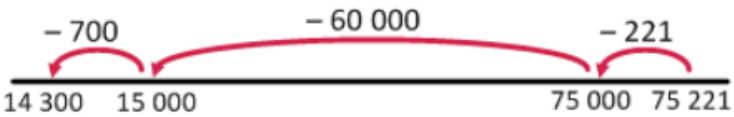
Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count back to a multiple of 10.



Calculate difference by "counting back"

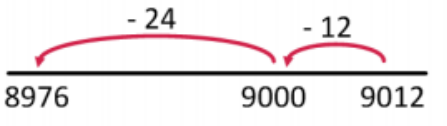
It is interesting to note that finding the difference is reversible. For example, the difference between 5 and 2 is the same as the difference between 2 and 5. This is not the case for other subtraction concepts. 75 221 – 14 300

Place the numbers either end of a numberline and work out the difference between them. Select efficient jumps.



Finding the difference is efficient when the numbers are close to each other:

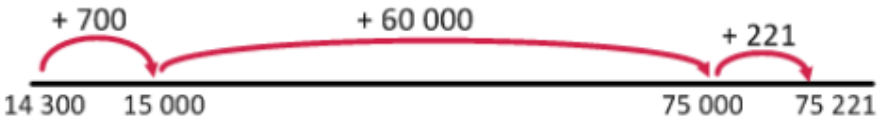
9012 – 8976



Calculate difference by "counting on"

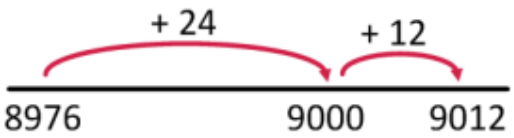
Addition strategies can be used to find difference.

75 221 – 14 300



Finding the difference is efficient when the numbers are close to each other

9012 – 8976



Y5

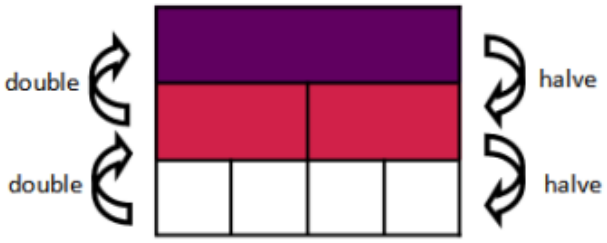
Multiplication

Y6

Multiply up to 4-digits by 1-digit (Y5) or 4-digits by 2-digits (Y6)

Doubling and halving

Pupils should experience doubling and halving larger and smaller numbers as they expand their understanding of the number system. Doubling and halving can then be used in larger calculations .

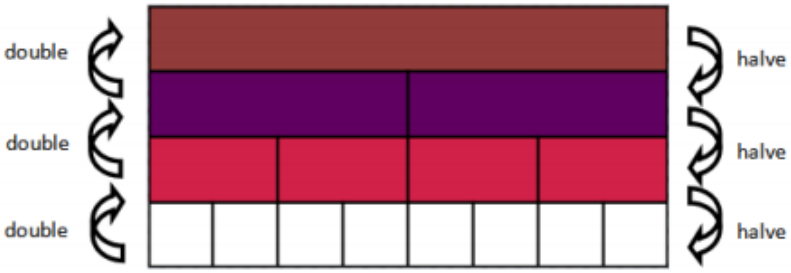


Multiply by 4 by doubling and doubling again

e.g. 16 × 4 = 32 × 2 = 64

Divide by 4 by halving and halving again

e.g. 104 ÷ 4 = 52 ÷ 2 = 26

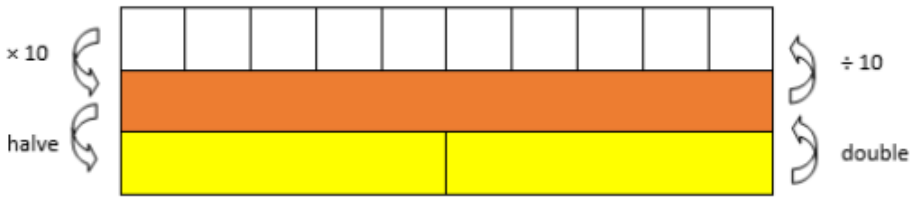


Multiply by 8 by doubling three times

e.g. 12 × 8 = 24 × 4 = 48 × 2 = 96

Divide by 8 by halving three times

e.g. 104 ÷ 8 = 52 ÷ 4 = 26 ÷ 2 = 13



Multiply by 5 by multiplying by 10 then halving,

e.g. 18 × 5 = 180 ÷ 2 = 90.

Divide by 5 by dividing by 10 and doubling,

e.g. 460 ÷ 5 = double 46 = 92

Y5

Multiplication

Multiply up to 4-digits by 1-digit (Y5) or 4-digits by 2-digits (Y6)

Y6

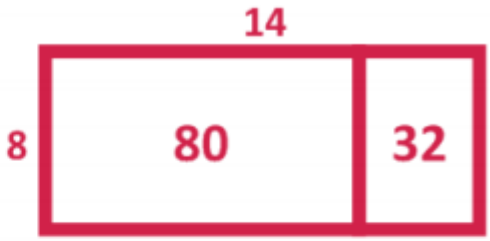
Multiply by partitioning one number and multiplying each part

Distributive law a x (b + c) = a x b + a x c

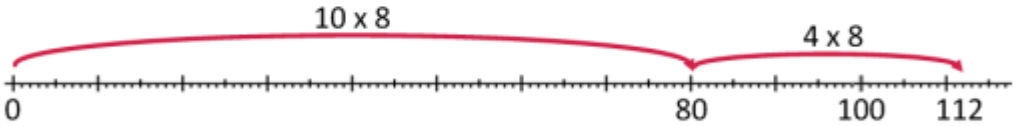
Build on pupils' understanding of arrays of counters to represent multiplication to see that area models can be a useful representation:

8 x 14 = 8 x 10 + 8 x 4

Cuisenaire rods to build arrays

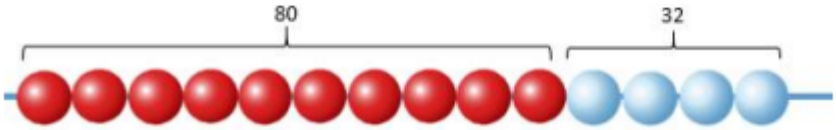


Represent with area model



Jottings on a number line

Bead string where each bead has a value of 8:



Using knowledge of factors

In Year 5 pupils are expected to be able to identify factor pairs and this knowledge can be used to calculate. Pupils will be using the commutative and associative laws of multiplication.

Commutative law a x b = b x a Associative law a x b x c = (a x b) x c = a x (b x c)

They should explore and compare the different options and choose the most efficient order to complete calculations.

Y5

Division

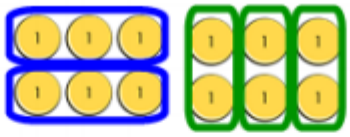
Y6

Divide up to 4-digits by 1-digit (Y5) or 4-digits by 2-digits (Y6)

Deriving facts from known facts

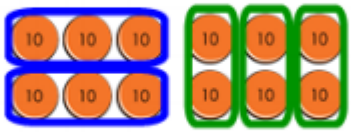
Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally. Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.

6 ÷ 2 = 3



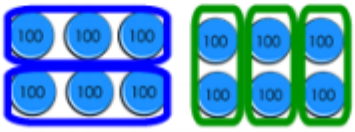
6 ÷ 3 = 2

60 ÷ 2 = 30
60 ÷ 30 = 2



60 ÷ 3 = 20
60 ÷ 20 = 3

600 ÷ 2 = 300
600 ÷ 300 = 2



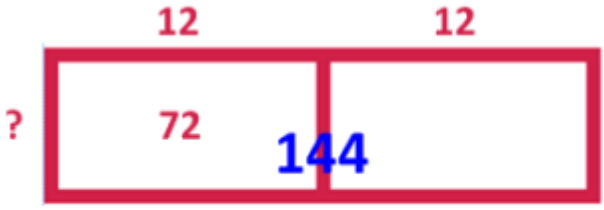
600 ÷ 3 = 200
600 ÷ 200 = 3

Using knowledge of factors to divide

Pupils explore this strategy when using repeated halving. 2 x 2 = 4 and so if you divide by 4 the same result can be achieved by dividing by two and then by two again .

144 ÷ 24

I know 2 and 12 are a factor pair of 24 and so I can divide by 2 and then by 12.



144 ÷ 2 ÷ 12

Y5

Division

Y6

Divide up to 4-digits by 1-digit (Y5) or 4-digits by 2-digits (Y6)

Using knowledge of multiples to divide

Using an area model to partition the whole into multiples of the divisor (the number you are dividing by).